ISSN 1936-5349 (print)
ISSN 1936-5357 (online)

HARVARD

JOHN M. OLIN CENTER FOR LAW, ECONOMICS, AND BUSINESS

LOYALTY DISCOUNTS AND NAKED EXCLUSION

Einer Elhauge

Discussion Paper No. 608

02/2008

Harvard Law School
Cambridge, MA 02138

LOYALTY DISCOUNTS AND NAKED EXCLUSION

By Professor Einer Elhauge
Harvard University*
February 15, 2008

ABSTRACT

Loyalty discounts are agreements to sell at a lower price to buyers who buy all or most of their purchases from the seller. This article proves that loyalty discounts create anticompetitive effects, not only because they can impair rival efficiency, but because loyalty discounts perversely discourage discounting even when they have no effect on rival efficiency. The essential reason, missed in prior work, is that firms using loyalty discounts have less incentive to compete for free buyers, because any price reduction to win sales to free buyers will, given the loyalty discount, also lower prices to loyal buyers. This in turn reduces the incentive of rivals to cut prices, because there will exist an above-cost price that rivals can charge to free buyers without being undercut by the firm using loyalty discounts. These anticompetitive effects occur even if buyers can breach or terminate commitments, and even if the loyalty conditions require no contractual commitments and less than 100% loyalty. Further, I prove that these anticompetitive effects are exacerbated if multiple sellers use loyalty discounts. None of the results depend on switching costs, market differentiation, imperfect competition, or the loyalty discount bundling contestable and incontestable demand. Contrary to commonly held views, I prove these anticompetitive effects exist even when: (1) the price with the loyalty discount is above cost, (2) the rival has higher costs than the firm using loyalty discounts, (3) the rival prices above its own costs, (4) buyers voluntarily agree to the conditions, and (5) the discount and foreclosure levels are low. I derive formulas for calculating the inflated price levels in each situation.

JEL Codes: C72, K21, L12, L40, L41, L42.

Keywords: Loyalty, Discounts, Rebates, Loyalty Discounts, Fidelity Rebates, Loyalty Rebates, Conditional Rebates, Conditional Discounts, Market-Share Discounts, Exclusionary Discounts, Exclusionary Rebates, Bundled Discounts, Mixed Bundling, Naked, Exclusion, Naked Exclusion, Exclusionary, Exclusive Dealing, Antitrust, Competition, Profit Sacrifice, Foreclosure, Cumulative Foreclosure, Aggregate Foreclosure, Terminability, Breach, Equally Efficient Rival, Economies of Scale, Loyalty Commitment, Incontestable Demand, Predation, Monopolization, Attempted Monopolization, Clayton Act, Sherman Act, Article 81, Article 82, Abuse of Dominance, Conditioned Sales, Exclusionary Conditions, Exclusionary Conduct, Predatory Conduct, Anticompetitive, Anticompetitive Conduct, Anticompetitive Practices, Anticompetitive Acts, Anticompetitive Agreements, Restraints of Trade, Rule of Reason.

---

* Petrie Professor of Law, Harvard Law School, 1575 Massachusetts Ave., Cambridge, MA 02138 (email: elhauge@law.harvard.edu).

Exclusionary agreements condition favorable terms on buyers restricting their purchases from rivals. The Chicago School has long asserted that such exclusionary agreements could never be anticompetitive because, if they were, the harm to buyers would exceed the benefits to buyers from agreeing, and thus buyers would not agree. (Posner, 2001; Bork, 1978). An important set of articles has proven that this Chicago School assertion is false.

One pair of seminal articles showed that, if buyers honor their exclusionary commitments, then a seller who makes discriminatory or sequential offers can get buyers to agree to anticompetitive exclusionary agreements that deprive rivals of economies of scale, even when buyers coordinate and Bertrand competition is assumed. (Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000), henceforth RRW-SW.) The essential reason is that each buyer's decision to agree does not consider the externality imposed on other buyers by the exclusionary agreement's contribution to the marketwide harm of excluding a rival that would lower prices for all buyers. RRW-SW also show that, if buyers cannot coordinate with each other, then even a seller that makes nondiscriminatory simultaneous offers can get buyers to agree to exclusionary commitments that harm them.[1] Buyer coordination is generally unlikely because antitrust law makes it illegal for rival buyers to agree on the terms they will accept, or even to exchange information about the terms being offered by specific sellers, with the violation subject to treble damages and possible criminal punishment.[2] In any event, because discriminatory or sequential offers are generally possible, it would seem a seller can usually overcome buyer coordination even if it were allowed.

However, another important recent article argues that, if buyers can breach their exclusionary commitments upon a payment of expectation damages equal to the

---

[1] Although nondiscriminatory simultaneous offers can theoretically produce a failure to agree as well, an excellent recent experimental study showed that noncoordinating buyers facing nondiscriminatory simultaneous offers agreed to anticompetitive exclusionary agreements 92% of the time. Landeo and Spier (2007). This was true even though the experiments used only two buyers, which should make the odds of rejection higher than typical because the greater the number of buyers, the less likely it is that any individual buyer's agreement will make a decisive difference to whether the marketwide foreclosure results, resulting in less buyer incentive to resist.

[2] Mandeville Island Farms v. American Crystal Sugar, 334 U.S. 219 (1948) (illegal for rival buyers to agree on terms they will pay); United States v. Container Corp., 393 U.S. 333 (1969) (illegal for rivals to exchange information on terms each is offering); United States v. United States Gypsum, 438 U.S. 422 (1978) (rival information exchange on terms each is offering is subject to possible criminal penalties).

difference between the monopoly price and the rival price, then (if we assume Bertrand competition) such a commitment cannot prevent a rival from inducing consumers to breach their exclusionary commitments. (Simpson & Wickelgren, 2007, henceforth S&W). They reason that breaching buyers will save an amount equal to expectation damages by shifting the purchases they would have made without breach to a rival that offers to sell at cost,[3] and in addition buyers would save the deadweight loss because they would buy a greater quantity when buying at cost. However, this article also shows that, if the buyers are not consumers, but rather are intermediate buyers who compete downstream in a competitive market, then sellers can get them to accept an anticompetitive exclusionary agreement in exchange for a small sidepayment, because the intermediate buyers externalize the anticompetitive harm onto downstream consumers.[4] They further prove that this latter point is true even if there are no relevant economies of scale.

Although these models are highly illuminating, in all of them the seller offers a form of exclusionary agreement one does not often observe in the real world. Namely, these models assume the seller offers a payment in period 1 for the buyer agreeing to buy exclusively from the seller in period 2 at whatever price the seller chooses to set in period 2. This price will be set at the monopoly level, which if a rival enters will be higher not only than the rival price, but also higher than the price the seller charges to nonexclusive buyers. That is, these models assume that exclusive dealing will lead to loyalty penalties, with sellers charging exclusive buyers a higher price than they charge to nonexclusive buyers. One does not often observe that in real markets.

What is commonly observed, and a very hot topic of antitrust debate recently, are loyalty discounts. With a loyalty discount, a seller agrees to charge loyal buyers a price that is lower than the price (often called the list price) that the seller charges to free purchasers. Sometimes, the agreements involve a buyer commitment of exclusivity that cannot be violated without committing contractual breach. Other times, such contracts are terminable by the buyer, and sometimes the buyer makes no commitment at all, but simply buys under a contract that sets one price if it complies with the loyalty condition and a higher price if it does not. The loyalty condition may require the buyer to buy 100% from the seller to get the discount, or instead some

[3] The Bertrand assumption is necessary to justify this premise that a rival facing substantial foreclosure would be able to instantly produce at scale sufficient to price at cost.

[4] This point had previously been made, without formal proof, in Elhauge (2003).

lower threshold, such as 80% or 90%.

This article analyzes such loyalty discounts, and proves that they raise prices above competitive levels for both loyal and free buyers, even if we assume Bertrand competition without economies of scale or without any impairment of rival efficiency. The essential reason, missed in prior work, is that firms using loyalty discounts have less incentive to compete for free buyers, because any price reduction to win sales to free buyers will, given the loyalty discount, also lower prices to loyal buyers. This in turn reduces the incentive of rivals to cut prices, because there will exist an above-cost price that rivals can charge to free buyers without being undercut by the firm using loyalty discounts. This is true even though I assume Bertrand competition in a homogeneous product for buyers who are ultimate consumers and thus cannot pass along any portion of the price increase. These adverse price effects are worsened if the exclusionary agreements do exclude rivals or impair rival efficiency.

I prove that these anticompetitive effects exist even if we assume buyers would breach loyalty commitments when the gains exceed expectation damages. Further, I go beyond that to prove that anticompetitive effects persist even if buyers make no commitments, and thus are free to violate the loyalty condition without paying any damages whenever they can get a better deal from the rival. Indeed, in such a case, the fact that accepting buyers never have to pay more than the rival would charge makes it even easier to show that buyers will agree to anticompetitive loyalty discounts, and thus prove that the equilibrium will produce anticompetitive results. I also prove these anticompetitive effects exist even when less than 100% loyalty is required to trigger the loyalty discounts.

I extend the analysis to cases where multiple firms offer loyalty discounts with commitments, and prove that this exacerbates the anticompetitive effects. The essential reason is that the resulting cumulative foreclosure leaves fewer uncommitted buyers available, and thus creates even less incentive for either firm to undercut uncommitted prices to get them, given that doing so reduce the committed prices of each. Cumulative foreclosure also makes it even more likely that other rivals will be unable to achieve economies of scale. Finally, when both firms offer loyalty discounts, the anticompetitive equilibria are even more likely and less vulnerable to defection.

I. The Model

Assume the market has N buyers, each of which have the same downward sloping demand function, $q = (1/N)(A - P)$, where q is the quantity demanded by each buyer, A is a constant, and P is the price the buyer pays. If all buyers pay the same price, the total quantity demanded $Q = qN = A - P$.[5] The market has two potential producers, the firm using loyalty discounts (who I will call the "user") and the rival, who produce identical products and have the same average cost function that depends on the quantity each produces, Q_i, and the same recurring fixed cost F, with $C(Q_i) = F/Q_i$ for $Q_i < Q^*$ and $C(Q_i) = F/Q^* = \boldsymbol{C}$ for all $Q_i \geq Q^*$. The minimum efficient scale is thus Q^*, and I assume the market is not a natural monopoly by assuming $Q^* < (1/2)(A - \boldsymbol{C})$. Given these assumptions, the competitive cost and price $= \boldsymbol{C} = F/Q^*$, the competitive output $= A - \boldsymbol{C}$, the monopoly output of $Q_m = (A - \boldsymbol{C})/2$, and the monopoly price of $P_m = (A + \boldsymbol{C})/2$.

In period 1, the user offers a loyalty discount agreement to buyers. I will begin with the assumption that accepting the loyalty discount commits buyers to buy 100% from the user and that buyers always comply with their commitments. Later, I extend the analysis to cases where (1) buyers commit but breach when that is profitable, (2) buyers make no contractual commitment, (3) less than 100% loyalty is required, and (4) both the user and rival offer loyalty discounts with commitments. In all these cases, I assume a loyalty discount agreement requires the user to charge $P_f - d$ to loyal buyers in period 3, where P_f is the price the user charges to buyers free of loyalty conditions and d is the loyalty discount. Thus, the loyalty discount commits the user to charge loyal buyers less than it charges free buyers. I assume buyer coordination is impossible, which is realistic given the legal penalties on it and the large number of buyers in many markets. The loyalty discount agreement is signed by S buyers. I will use θ to denote S/N, the share of buyers that agreed to loyalty discounts.

In period 2, the rival decides whether to incur recurring fixed costs F in order to have the capacity to make a product. The prior papers considered only whether the rival would decide to enter, which is the special case where the rival has no capacity and decides whether to build any, but I generalize the issue to include the rival that has existing capacity and is deciding whether to incur the recurring fixed costs to maintain or expand it.

[5] The analysis extends to any linear demand curve $Q = A - BP$ because one could convert that into an equation that takes the form $Q = A - P$ by using a measure of units that makes $B = 1$.

In period 3, I adopt the assumption, like prior papers, that if the rival enters, the user and rival engage in Bertrand competition. The Bertrand model is extreme because it unrealistically assumes that output is infinitely and instantly expandable, that there is no product differentiation or switching costs, and that competition is a single period game so that firms need not fear reactions in subsequent periods, all of which results in the "strained" conclusion that (without loyalty discounts) a duopoly will produce the same prices as a perfectly competitive market.[6] Nonetheless, I here assume a Bertrand model assumption for two reasons. First, it biases the case against finding anticompetitive effects.[7] Second, it makes it easier to compare the conclusions here with those reached in prior papers about naked exclusion because they used Bertrand models.

Given the assumption of Bertrand competition, if $S = 0$, then both the rival and user will set prices equal to $\boldsymbol{C}$. This is thus the but-for baseline without any loyalty discounts.

II. If Buyers Honor 100% Loyalty Commitments

I begin with the case where the loyalty discount agreement requires buyers to commit to make 100% of their purchases from the user. Like the prior papers, I first analyze the period 3 outcomes (here under the alternative assumptions that the loyalty discounts do not or do impair rival efficiency), then consider the effects of those possible outcomes on rival production in stage 2 and on buyer willingness to agree in stage 1.

<u>a. No Loss of Rival Efficiency.</u> Take first the case where the uncommitted market is large enough to allow the rival to operate at minimum efficient scale if it can win all uncommitted buyers, that is $(N\text{-}S)q(\boldsymbol{C}) \geq Q^*$. (The analysis that follows also applies when there are no economies of scale and incremental costs $\boldsymbol{C}$ are constant.) Given the Bertrand assumptions, the rival will thus produce and has to decide what price to charge. For any rival price, P_r, that the rival chooses, the user has two options. First, it can deprive its rival of all sales by lowering its uncommitted price to some infinitesimal amount less than the rival price, $P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to $N\text{-}S$ buyers

---

[6] Tirole (1988), at 211.

[7] *Id.* at 212. The reasons will be explained in the Implications section below.

and P_r - ε - d to S buyers. Second, it can concede all uncommitted buyers, but still make all sales to the committed buyers, in which case it will maximize profits by charging $P_f - d = P_m$ to these S buyers.

The rival earns zero profits from the first option or from pricing at $P_r = \boldsymbol{C}$. Thus, the rival will want to set $P_r > \boldsymbol{C}$ but sufficiently low that the user finds it more profitable to sell to the committed buyers at the monopoly price, rather than try to undercut the rival price for uncommitted buyers. Ignoring the ε, since it is infinitesimally small, this condition is met when:

$$(P_m\text{-}\boldsymbol{C})(S/N)(A\text{-}P_m) > (P_r\text{-}\boldsymbol{C})((N\text{-}S)/N)(A\text{-}P_r) + (P_r\text{-}d\text{-}\boldsymbol{C})(S/N)(A\text{-}P_r\text{+}d).$$

Given that $A = 2\mathrm{P}_m - \mathrm{C}$, and $\theta = S/N$, this can be rearranged as:

$$P_r^{\,2} - 2(P_m+\theta d)P_r + 2\boldsymbol{C}P_m - \boldsymbol{C}^2 + 2\theta dP_m + \theta d^2 + \theta \cdot (P_m\text{-}\boldsymbol{C})^2 > 0.$$

The Appendix proves that this inequality will be satisfied as long as the rival charges no more than

$$P_r^* = P_m + \theta d - [(1\text{-}\theta)((P_m\text{-}\boldsymbol{C})^2 - \theta d^2)]^{½},$$

and that P_r^* is always above cost and more profitable for the rival than any alternative rival price as long as $P_r^* < P_m$. If $P_r^* \geq P_m$, then the rival will find it more profitable to charge P_m, which the Appendix proves will be true if $P_m\text{-}\boldsymbol{C} \leq [\theta/(1\text{-}\theta)]^{½}d$. This thus proves Proposition 1.

Proposition 1. Suppose there are no economies of scale or the rival produces enough to reach its minimum efficient scale, and the rival and user engage in Bertrand competition. If the loyalty discounts have commitments with which buyers comply, then the user will make all sales to committed buyers at $\boldsymbol{P_m}$. The rival make all sales to uncommitted buyers at:

(1) $\boldsymbol{P_m}$ if $\boldsymbol{P_m\text{-}C} \leq [\boldsymbol{\theta}/(1\text{-}\boldsymbol{\theta})]^{½}\boldsymbol{d}$, or

(2) $\boldsymbol{P_r^* = P_m + \theta d} - [(1\text{-}\boldsymbol{\theta})((\boldsymbol{P_m\text{-}C})^2 - \boldsymbol{\theta d^2})]^{½}$ if $\boldsymbol{P_m\text{-}C} > [\boldsymbol{\theta}/(1\text{-}\boldsymbol{\theta})]^{½}\boldsymbol{d}$.

All these prices will exceed the but-for competitive level, *C,* which would have prevailed without the loyalty discounts on the same market assumptions.

This is a subgame perfect Nash equilibrium. The rival will not charge $P_r > P_r^*$ because that would cause the user to charge $P_f < P_r$ to uncommitted buyers and lower rival sales and profits to zero. If $P_r^* > P_m$, the rival will charge P_m and would not charge any less because that would result in lower profits. If $P_r^* \leq P_m$, the rival will charge P_r^* because any lower price will bring it further below the profit-maximizing level and thus earn it less money. Given that the rival is charging no more than P_r^*, the user will not have any incentives to charge committed buyers less than P_m because it cannot undercut the rival price to uncommitted buyers without resulting in lower overall profits.

In short, loyalty discounts cause an effective market division, where both the user and the rival price above competitive levels without any agreement or coordination. When the foreclosure share and discount are sufficiently large, both price at monopoly levels. Otherwise, the user prices at monopoly levels, while the rival prices at a submonopoly level that is still well above the competitive level.

Strikingly, anticompetitive prices results even if the discount level is zero, which would be true if a particular loyalty discount did not assure any particular gap between committed and uncommitted prices, but simply guaranteed that the user would never charge committed buyers more than uncommitted buyers. Then the rival will charge $P_r{}^* = P_m - [(1\text{-}\theta)((P_m\text{-}C)^2]^{½} = (1\text{-} (1\text{-}\theta)^{½})P_m + (1\text{-}\theta)^{½}\boldsymbol{C}$, which always exceeds competitive level $\boldsymbol{C}$ if $P_m > \boldsymbol{C}$. The reason for this result is that even this weak assurance to committed buyers means that the user will lose profits from the committed buyers if it cuts prices below this level to match the rival on uncommitted buyers.

<u>b. Rival Efficiency Impaired.</u> Now suppose $(N\text{-}S)q(\boldsymbol{C}) < Q^*$. That is, the uncommitted buyers do not buy enough to allow the rival to achieve its minimum efficient scale, even if it wins all the uncommitted buyers and prices at cost. The rival cannot charge any more than $P_r{}^*$ without the user undercutting its price to uncommitted buyers, resulting in zero profits to the rival. But it also cannot charge any less than $C_r = F/Q_r = F/[(1\text{-}\theta)(A\text{-}C_r)]$, which means

$$C_r^{\,2} - (2P_m\text{-}\boldsymbol{C})C_r + F/(1\text{-}\theta) = 0,$$

the lowest quadratic solution to which is

$$C_r = P_m - \boldsymbol{C}/2 - [(P_m\text{-}\boldsymbol{C}/2)^2 - F/(1\text{-}\theta)]^{½}$$

Thus, the rival will not produce if

$$P_r{}^* < P_m - \boldsymbol{C}/2 - [(P_m\text{-}\boldsymbol{C}/2)^2 - F/(1\text{-}\theta)]^{½}$$

If the rival does not produce, then the user will maximize its total profits from committed and uncommitted buyers by maximizing the following:

$$(P_f\text{-}\ d - \boldsymbol{C})(S/N)(A - P_f + d) + (P_f\text{-}\ \boldsymbol{C})((N\text{-}S)/N)(A - P_f).$$

Taking the derivative with respect to P_f, this is maximized when

$$(A - P_f + d)\theta - (P_f\text{-}\ d - \boldsymbol{C})\theta + (A - P_f)(1\text{-}\theta) - (P_f - \boldsymbol{C})(1\text{-}\theta) = 0,$$

which boils down to $P_f = P_m + \theta d$. The price the user charges committed buyers will then be $P_f - d = P_m - (1\text{-}\theta)d$.

If $P_r{*} > P_m - \boldsymbol{C}/2 - [(P_m-\boldsymbol{C}/2)^2 - F/(1-\theta)]^{1/2}$, then the rival charges up to $P_r{*}$ and the user charges P_m. If the profit-maximizing price the rival can charge uncommitted buyers is less than $P_r{*}$, then the rival will price to maximize $(P_r - F/Q_r)(Q_r)$, which is the same as $-(1-\theta)P_r^2 + (1-\theta)AP_r - F$. Taking the first and second derivative shows that profits are maximized at $P_r = A/2 = P_m - \boldsymbol{C}/2$. Thus, the rival will charge the lesser of $P_r{*}$ or $P_m - \boldsymbol{C}/2$.[8] This proves Proposition 2.

Proposition 2. Suppose buyers comply with loyalty commitments, these commitments foreclose enough of the market to prevent the rival from reaching its minimum efficient scale, and the rival and user engage in Bertrand competition. Then

a. if $P_r{*} < P_m - C/2 - [(P_m-C/2)^2 - F/(1-\theta)]^{1/2}$, the rival will not produce, and the user will sell to uncommitted buyers at $P_m + \theta d$ and to committed buyers at $P_m - (1-\theta)d$, for an average price of P_m to all buyers.

b. if $P_r{*} > P_m - C/2 - [(P_m-C/2)^2 - F/(1-\theta)]^{1/2}$, the user will sell to committed buyers at P_m, and the rival will sell to uncommitted buyers at the smaller of $P_m - C/2$ or $P_r{*} = P_m + \theta d - [(1-\theta)((P_m-C)^2 - \theta d^2)]^{1/2}$.

All these prices will exceed the but-for competitive level C that would have prevailed without the loyalty discounts.

In short, if the foreclosure is significant enough, the rival cannot profitably produce on the market, creating a monopoly that would not have existed in the but-for world. Even if the foreclosure is lower than that, it will result in the user pricing at monopoly levels and the rival pricing at levels above the but-for competitive level, as well as creating productive inefficiency because the rival is producing at higher costs than it would have had in the but-for world.

<u>c. Will Buyers Accept Simultaneous Nondiscriminatory Offers?</u> Assume the user makes a simultaneous nondiscriminatory offer to charge $P_f - d$ to any buyer who will commit to buy exclusively from the user. Each buyer i will reason that there are four different possibilities.

One possibility is that buyers believe enough other buyers will accept the loyalty discount that the rival will not produce. If the first possibility arises, all buyers will pay d less per unit if they accepted the offer, thus each has incentives to accept the

[8] $P_r{*}$ will be the lower figure if $(1-\theta)(P_m-\boldsymbol{C})^2 > \theta d^2 + \theta d\boldsymbol{C} + \boldsymbol{C}^2/4$.

offer. This is a subgame perfect Nash equilibrium because no individual buyer has incentives to reject, even though the collective result of them all accepting is to raise prices.

A second possibility is that buyers believe enough other buyers will reject the loyalty discount that the rival can produce at its minimum efficient scale, but also believe that the foreclosure level will be high enough that P_m-$\boldsymbol{C} < [\theta/(1-\theta)]^{1/2}d$. Then the rival will charge P_m and buyers who reject the loyalty discounts will thus pay the same as those who accepted it. Buyers with this belief will be indifferent, and thus willing to agree to the loyalty discount for any infinitesimally small sidepayment ε. If increasing d increases the foreclosure share, the user may well be able to secure this result by raising d to high levels. (Notice there is no reason for d to be limited to P_m-$\boldsymbol{C}$ because d is merely the difference between the committed and uncommitted price.) If obtained, this possibility is a subgame perfect Nash equilibrium.

A third possibility is that buyers believe enough other buyers will reject the loyalty discount that the rival can produce at its minimum efficient scale and that P_m-$\boldsymbol{C} >$ $[\theta/(1-\theta)]^{1/2}d$. Then the rival will charge $P_r^* < P_m$, and buyers who reject the loyalty discounts will pay P_m - P_r^* less than those who accepted it. Buyers will thus have incentives to reject the loyalty discount, unless the user can offer them a sidepayment that exceeds the expected losses to agreeing buyers. The gain to the user from the agreement of each buyer will be $(P_m$-$\boldsymbol{C})(Q_m/N)$. The loss to each agreeing buyer will be the difference between their consumer surpluses at P_r^* and P_m, which will be $(1/2)(A$-$P_r^*)Q_r/\mathrm{N}$ - $(1/2)(A$-$P_m)Q_m/N$. The Appendix proves Lemma 1, that the user can offer a sidepayment that induces buyers with this expectation to agree as long as $P_r^* \geq .27P_m + 0.73\boldsymbol{C}$.

Lemma 1. If a buyer believes enough other buyers will reject the loyalty discount that the rival will enter and price below the monopoly level, the buyer can be induced to accept the loyalty discount for a sidepayment as long as $P_r^* \geq .27P_m + 0.73\boldsymbol{C}$.

A fourth possibility is that buyers believe the number of buyers who accept the loyalty discounts will be small enough that the rival does produce, but large enough that rival efficiency is impaired. Then the rival will charge the smaller of P_r^* and P_m-$\boldsymbol{C}/2$. Buyer who reject the loyalty discount will thus be better off by at least $\boldsymbol{C}/2$ and perhaps a larger figure of P_m-P_r^*. Lemma 1 would determine whether a profitable sidepayment could be made given P_r^*. Further, since we know the maximum P_r^* is

P_m-$\boldsymbol{C}/2$ we also know a sidepayment is possible only if P_m-$\boldsymbol{C}/2 > .27P_m + 0.73\boldsymbol{C}$, which means P_m must be $> 1.69\boldsymbol{C}$, so that the monopoly profit margin $(P_m$-$\boldsymbol{C})/P_m$ must be at least 40%.

In sum, there is always a subgame perfect Nash equilibrium where buyers accept harmful loyalty discounts if they believe enough other buyers will accept that (1) the rival will not produce or (2) the rival will enter and charge a monopoly price. In these cases, both the buyers who accept and reject the loyalty discounts will be harmed relative to the but-for world where they would have paid $\boldsymbol{C}$. If buyers do not hold either of the above beliefs, then there can also be a subgame perfect Nash equilibrium where buyers accept harmful loyalty discounts if they expect $P_r^* \geq .27P_m + 0.73\boldsymbol{C}$, because the user can pay each buyer enough to make acceptance profitable for each accepting buyer and the user. Rejecting buyers will be harmed relative to the but-for world because they will pay P_r^* rather than $\boldsymbol{C}$. Accepting buyers as a group will also be harmed because, but for the loyalty discounts, they would have paid $\boldsymbol{C}$ and enjoyed more consumer welfare, but they are individually better off accepting because, given the loyalty discounts, the alternative to accepting is paying P_r^*.

Buyers might, of course, have mixed expectations. Suppose, for example, each buyer believes that each of the four possibilities is equally likely. Then the buyers will require no more than half the sidepayment suggested by possibilities three and four. They might even require no sidepayment at all if the discount is sufficiently higher than the difference between P_m and expected P_r^*, so that the expected gain from accepting under the first possibility exceeds any loss from accepting under the third and fourth possibilities. Now suppose buyers put different odds on each of the possibilities. The higher the odds they put on the first and second possibilities, the lower the sidepayment they would require. If they put sufficiently highly odds on the first possibility, none of them will require any sidepayment at all.

Importantly, the user can profit from a simultaneous nondiscriminatory offer without any sidepayment even with a low buyer acceptance rate. Suppose, for example, there are no economies of scale. Then if *any* buyers accept, offering the loyalty discount is profitable for the user because it earns greater profits than in the but-for world as long as $\theta > 0$. Likewise, making a simultaneous nondiscriminatory offer will always be profitable for the user as long as it can achieve its minimum efficient scale from the foreclosed buyers. Indeed, even if the loyalty discounts drive the user below its minimum efficient scale, loyalty discounts without any sidepayments will be profitable whenever the profits the user earns from the foreclosed buyers exceed zero.

Using P_u and Q_u to denote the user's price and quantity, this is so whenever $(P_u - F/Q_u)(Q_u) > 0$, which is the same as $-\theta P_u^2 + (2P_m-\boldsymbol{C})\theta P_u - F > 0$. Taking the first and second derivative shows that, below its minimum efficient scale, the user will maximize profits by pricing to foreclosed buyers at $P_u = P_m - \boldsymbol{C}/2$. Plugging into the above inequality shows that profits at this price will exceed zero as long as $\theta > F/(P_m-\boldsymbol{C}/2)^2$.

<u>d. Will Buyers Accept Sequential Offers?</u> Now consider the possibility that the user can make sequential offers, and buyers differ in their beliefs, with some set holding the beliefs indicated by each of the different possibilities. As the above suggests, first making a simultaneous nondiscriminatory offer without any sidepayments often has no downside. Suppose some share of buyers θ held the beliefs indicated by possibilities one or two, and thus accepted the simultaneous nondiscriminatory offer.

Then the user makes a second round of offers to the other buyers. If θ turned out to be large enough that possibilities one or two were in fact realized, then the other buyers will accept a second round offer that is simultaneous and nondiscriminatory. If θ was not that large, then even if the second round offer could be made to only one buyer, it would will still find it profitable to accept the second round offer as long as θ is large enough that $P_r^* \geq .27P_m + 0.73\boldsymbol{C}$. In fact, there will be multiple buyers, and each buyer knows that if it rejects the second round offer, the offer can be made sequentially to the other buyers, and that as more buyers accept that will raise P_r^* and lower the required sidepayment in later rounds, until no sidepayment is required at all. This will often make it possible to get such second round buyers to agree for no sidepayment at all, for reasons parallel to those explained in RRW-SW for sequential offers.

III. If Buyers Breach Loyalty Commitments When Profitable

Now consider the possibility, raised by S&W, that buyers would breach their exclusionary commitments if the gains from doing so exceed their contract expectation damages. This assumption is actually quite debatable. As they acknowledge in their thoughtful article, reputational considerations and legal costs will often deter breach in such a case. Indeed, some contracts scholarship indicates that reputational sanctions are often more important in securing compliance than legal penalties.[9]

More important, legal penalties for breach of contract are not limited to expectation damages. S&W assume otherwise because of the contract rule barring penalty clauses that set damages higher than expectation damages, but their assumption ignores forfeiture penalties. Under standard contract law, a buyer's intentional breach of an exclusivity commitment would allow any seller to decline to fulfil any of its own contractual commitments.[10] If the seller wished to remove any doubt about the matter, it could simply make its duties explicitly conditional on the buyer honoring its exclusivity commitment. Thus, in addition to expectation damages, a buyer will suffer the harm of forfeiting the value of its other contract rights. If the relevant contract includes products other than the one in question, the penalty of forfeiting these contractual rights could be enormous. This may help explain why loyalty discounts are often bundled with loyalty discounts on other products.

Even if the contract is limited to the particular product, breach can also allow the seller to suspend duties as to past sales, such as a duty to repair or pay rebates on past sales. Those can create large penalties that exceed expectation damages. Indeed, it is relatively easy to evade the ban on penalty clauses by reframing them as conditional bonuses or rebates. For example, suppose expectation damages of X per unit would not deter breach of the exclusivity commitment, but $2X$ would. If the contract just had a clause making breach punishable by $2X$, then S&W would be right that this would violate the ban on penalty clauses. But the ban would not prevent the user from charging P_m+2X with a rebate of $2X$ to buyers who comply with the exclusivity condition.[11] Then buyers would comply because failure to do so would result in a loss of $2X$, and since compliant buyers would on net pay P_m they would behave just like committed buyers who comply under Propositions 1 and 2. Thus, a user could always evade the obstacle observed by S&W by having a rebate conditional

[9] *See* Schwartz & Scott (2003), at 557.

[10] If the market involves the sale of goods, this is true even for unintentional non-material breaches under the perfect tender rule. *See* Farnsworth, 551-52 (2004). For non-goods, a seller has the right to suspend performance only if the breach is material, but intentionality itself likely makes a breach material. *Id.* at 550-551. Even if the breach were unintentional, breaching a central commitment like exclusivity would likely be deemed material.

[11] Because the rebate is not a "penalty" but a bonus, it does not violate the ban on penalty clauses. Nor would courts review whether the rebate exceeds the value of performance because another doctrine prohibits inquiry into the adequacy of consideration for a promise.

on compliance with the exclusivity condition, as long as the rebate exceeds the consumer welfare gain from breaching the exclusivity commitment. This may help explain why loyalty rebates are often used instead of, or in addition to, loyalty discounts.

However, I need not rely on those additional reasons to expect compliance with commitments, because it turns out that expectation damages will alone suffice to deter breach in the cases where sellers can obtain loyalty discounts with commitments. S&W conclude otherwise for buyers who are ultimate consumers. However, their conclusion depends critically on the assumption that the rival will enter at a price equal to cost. If the rival does so, then consuming buyers who switch to the rival will have to pay contract damages of P_m - $\boldsymbol{C}$ on every purchase they would have made from the user, which will be offset by an equivalent gain of P_m - $\boldsymbol{C}$ in lower prices on every such purchase, and in addition buyers will gain the deadweight loss they otherwise would have suffered because buying at P_m would cause them to buy less than the efficient amount.

The reason S&W's analysis is inapplicable here is that the above shows that, given loyalty commitments, the rival will price at $P_r^* > \boldsymbol{C}$ even under the extreme assumption of Bertrand competition.[12] In the possibilities where $P_r = P_m$, there will be no gain to buyers from breaching, and thus expectation damages will clearly deter breach. In the possibilities where the rival charges $P_r^* < P_m$, we can determine when expectation damages will exceed buyer gains from breach by using the sidepayment analysis above. Because that analysis showed when an agreement creates user gains that exceed individual buyer losses, it also shows when breach of an agreement creates expectation damages to the user that exceed individual buyer gains from breach. Thus, expectation damages will make breach unprofitable whenever $P_r^* \geq .27P_m + 0.73\boldsymbol{C}$. Because that is the condition to get the last buyer to agree to loyalty discounts in the first place, it should be met for any set of loyalty discounts that actually exist.

[12] Likewise, S&W's conclusions would not hold if we more realistically assumed imperfect competition or lags in growing rival output, which would also make the rival price above ***C.***

IV. If Loyalty Discounts Require No Commitments and Less Than 100% Loyalty

Now consider the case where the loyalty discounts reflect conditions without any contractual commitments or rebates. That is, assume the buyers who accept the discounts do not promise to buy only from the user, but rather agree only that exclusivity will be a condition of getting discounts. At any time, the buyers can buy from the rival without incurring contractual liability or lost rebates, as long as they are willing to forgo future loyalty discounts. Even then, it turns out the effects of the loyalty discounts are anticompetitive.

a. Unconditioned Market Suffices to Achieve Minimum Efficient Scale. Take first the case where either (1) there are no economies of scale or (2) selling to buyers who have not agreed to the condition suffices to obtain all economies of scale, that is $(N\text{-}S)q(\boldsymbol{C}) \geq Q^*$. We must first consider how to apply the Bertrand model to the situation. Generally the Bertrand model assumes the price choices are simultaneous because it assumes the two firms are equally situated as to the buyers, and the above used that assumption because in the model the two firms are equally situated as to the relevant uncommitted buyers. However, here there is also the possibility that the rival can compete for buyers who are under contract with the user. Because these buyers are under contract with the user, it seems implausible they would switch without giving the user a chance to undercut any price the rival is offering. It thus makes sense to assume the rival must first offer them a better price, and that the user will have a chance to respond before the buyer switches. So I will begin by assuming that the existence of the contracts means the rival sets a price first, though later I explore the other possibilities that the user sets prices first or they set prices simultaneously.

For any rival price, P_r, greater than $\boldsymbol{C}$, the user has two options. First, it can deprive its rival of all sales by lowering its unconditioned price to some infinitesimal amount less than the rival price, P_r - ε, thus earning P_r - ε to $N\text{-}S$ buyers and P_r - ε - d to S buyers. Second, it can concede all unconditioned buyers, but still make all sales to the conditioned buyers, in which case it will maximize profits by charging $P_f - d = P_r -$ ε to these S buyers.

As above, the rival will want to set P_r sufficiently low to trigger the second response by the user because otherwise the rival will earn zero profits. This condition is met when:

$$(P_r\text{-}\boldsymbol{C})(S/N)(A\text{-}P_r) > (P_r\text{-}\boldsymbol{C})((N\text{-}S)/N)(A\text{-}P_r) + (P_r\text{-}d\text{-}\boldsymbol{C})(S/N)(A\text{-}P_r\text{+}d),$$

which boils down to

$$(1\text{-}\theta)P_r^2 - 2[(1\text{-}\theta)P_m + \theta d]P_r + (1\text{-}\theta)2\boldsymbol{C}P_m - (1\text{-}\theta)\boldsymbol{C}^2 + 2\theta dP_m + \theta d^2 > 0$$

The Appendix proves this will be true as long as the rival charges no more than

$$P_r^{**} = P_m + d\theta/(1\text{-}\theta) - [(P_m\text{-}\boldsymbol{C})^2 + \theta d^2(2\theta\text{-}1)/(1\text{-}\theta)^2]^{1/2}.$$

and that P_r^{**} is always above cost and more profitable for the rival than any alternative rival price as long as $P_r^{**} < P_m$. If $P_r^{**} \geq P_m$, then the rival will find it more profitable to charge P_m, which the Appendix proves will be true if $P_m\text{-}\boldsymbol{C} \leq [\theta/(1\text{-}\theta)]^{1/2}d$. This thus proves Proposition 3.

Proposition 3. Suppose there are no economies of scale or the unconditioned market is large enough for the rival to reach its minimum efficient scale, the user uses loyalty discounts that require no buyer commitment, and the rival and user engage in Bertrand competition, modified because the loyalty discounts mean the user can respond after the rival offers a price. Then:

a. If $P_m\text{-}C \leq [\theta/(1\text{-}\theta)]^{1/2}d$, the rival will sell to unconditioned buyers at P_m and the user will sell to conditioned buyers at $P_m - \varepsilon$.

b. If $P_m\text{-}C > [\theta/(1\text{-}\theta)]^{1/2}d$, the rival will sell to unconditioned buyers at $P_r^{**} = P_m + d\theta/(1\text{-}\theta) - [(P_m\text{-}C)^2 + \theta d^2(2\theta\text{-}1)/(1\text{-}\theta)^2]^{1/2}$ and the user will sell to conditioned buyers at $P_r^{**} - \varepsilon$.

All these prices will exceed the but-for competitive level, *C*, which would have prevailed without the loyalty discounts on the same market assumptions.

Given the assumptions, this is a subgame perfect Nash equilibrium. The rival will not charge $P_r > P_r^{**}$ because that would cause the user to charge $P_f < P_r$ to unconditioned buyers and lower rival sales and profits to zero. If $P_r^{**} > P_m$, the rival and user will respectively charge P_m to unconditioned buyers and $P_m - \varepsilon$ to conditioned buyers, and would not charge any less because that would result in lower profits. If $P_r^{**} \leq P_m$, the rival will charge P_r^{**} because any lower price will bring it further below the profit-maximizing level and thus earn it less money. Given that the rival is charging no more than P_r^{**}, the user will not have any incentives to charge conditioned buyers less than $P_r^{**} - \varepsilon$ because it cannot undercut the rival price to unconditioned buyers without resulting in lower overall profits.

If we assumed the user picked price first, then the rival could undercut a user price of $P_r^{**} - \varepsilon$ to the conditioned buyers. To avoid this, the user would thus want to pick the highest price, P_x, that is low enough that the rival finds it less profitable to sell to all buyers at $P_x - \varepsilon$ than to sell at P_r^{**} to just the unconditioned buyers. This will be true if the user picks the highest price that satisfies:

$$(1\text{-}\theta)(P_r^{**}\text{-}\boldsymbol{C})(A\text{-}P_r^{**}) > (P_x\text{-}\boldsymbol{C})(A\text{-}P_x)$$

The solution to which is

$$P_x = P_m - [P_m^2 - 2\theta \boldsymbol{C} P_m + \theta \boldsymbol{C}^2 + (1-\theta)(P_r**)^2 - 2(1-\theta)P_m P_r**]^{1/2},$$

which one can show is always greater than ***C.***

If one instead assumes simultaneous choice, the situation gets complicated. The rival knows that if the user picks a price to conditioned buyers $\leq P_x$, then its best price is P_r**. But if the rival picks P_r**, the best price for the user to charge conditioned buyers is P_r** - ε, in which case the rival should pick P_r** - 2ε. A Nash equilibrium thus seems to require a mixed strategy. However, the price to conditioned buyers will never be less than P_x or greater than P_r**. Thus, whether we assume simultaneous choices or sequential ones, the prices will at least be P_x and up to P_r**, all of which will exceed but-for competitive levels.

<u>b. Unconditioned Market Does Not Suffice to Achieve Minimum Efficient Scale.</u> Now suppose the unconditioned buyers do not buy enough to allow the rival to achieve its minimum efficient scale. As above, I assume the rival picks its price first. (If one assumes the user picks first, one can substitute P_x for P_r** below.) Given the analysis above, the rival can profitably restrict itself to the unconditioned buyers if $P_r** > P_m - \boldsymbol{C}/2 - [(P_m - \boldsymbol{C}/2)^2 - F/(1-\theta)]^{1/2}$. If so, the rival will sell to unconditioned buyers for the lesser of P_r** or $P_m - \boldsymbol{C}/2$, and the user will sell to the conditioned buyers for the lesser of P_r** - ε or $P_m - \boldsymbol{C}/2$ - ε.

If $P_r** < P_m - \boldsymbol{C}/2 - [(P_m - \boldsymbol{C}/2)^2 - F/(1-\theta)]^{1/2}$, then the rival cannot profitably restrict itself to selling to the unconditioned buyers. The rival will thus have to set a price low enough that the user would not have incentives to undercut it even as to conditioned buyers. The only price that satisfies this condition is ***C.*** Thus, under these assumptions, the rival will price at ***C*** and make all sales to unconditioned buyers. Assuming they split sales to conditioned prices at the same price, each will make half the sales to conditioned buyers. This proves Proposition 4.

Proposition 4. Suppose the user uses loyalty discounts that require no buyer commitment, the unconditioned market is not large enough for the rival to reach its minimum efficient scale, and the rival and user engage in Bertrand competition, but with the rival picking price first. Then

> **a. if $P_r** > P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}$, the rival will sell to unconditioned buyers for the lesser of P_r** or $P_m - C/2$, and the user will sell to the conditioned buyers for the lesser of P_r** - ε or $P_m - C/2$ - ε. All these prices will exceed but-for competitive level *C* that would have**

prevailed without the loyalty discounts.

b. if P_r $< P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}$, then the user will sell at C to half the conditioned buyers, and the rival will sell at C to the other half of the conditioned buyers and to all the unconditioned buyers.**

c. Will Buyers Accept? When loyalty discounts do not require commitments, buyers will always accept because they are never worse off doing so. This is because under every scenario described in Propositions 3 and 4, buyers are always better off having agreed to the loyalty discount, because the user price to conditioned buyers is always below or equal to the rival price to unconditioned buyers. Thus all buyers have incentives to accept.

If there are no economies of scale, then the user will want to keep offering loyalty discounts until almost all buyers accept because then it gets to price at P_m to more and more buyers. However, if there are economies of scale, then the user will want to stop offering loyalty discounts before θ rises to a level that drives P_r^{**} below $P_m - C/2 - [(P_m - C/2)^2 - F/(1-\theta)]^{1/2}$ because that would trigger a price war that drives prices for both down to costs.

Thus, when loyalty discounts do not require commitments, buyers will always accept them, and the user should always be able to offer enough loyalty discount agreements to make the prices of both the user and rival greater than their but-for levels.

d. Thresholds Less Than 100%. Now consider the possibility that the loyalty condition does not specify 100% exclusivity, but some threshold percentage $T < 1$ of purchases from the user. This does not change any of the analysis in the case of conditions without commitments. The reason is that the buyers who meet this threshold T will pay $P_f - d$ on all their purchases from the user, and since that is always less than P_r, the compliant buyers will make all their purchases from the user.

In the case of loyalty commitments with sub-100% thresholds, the analysis is more complicated because now the user has three options. First, it can deprive its rival of all sales by lowering its uncommitted price to $P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to N-S buyers and $P_r - \varepsilon - d$ to S buyers. Second, it can concede all uncommitted buyers (N-S), but still make all sales to the committed buyers, by lowering its committed price to $P_f - d = P_r - \varepsilon$, thus earning $P_r - \varepsilon$ to S buyers. Third, it can concede all uncommitted purchases (N-ST), and just sell T times the quantity purchased by committed buyers, by keeping $P_f - d = P_{m.}$ The first two are the same as the two options with 100%

loyalty conditions without commitments. Thus, the rival can always price at least at the levels indicated in Propositions 3 and 4 without triggering the first option. However, sometimes the user will find the third option more profitable than the second at those prices, in which case the rival faces less of a constraint and can price somewhat higher than in Propositions 3 and 4.[13]

V. When Multiple Firms Use Loyalty Discounts With Commitments

Now suppose a case where loyalty discounts are used by multiple firms. Take the case where there are no economies of scale or both firms achieve them, and the loyalty discounts require commitment with which buyers comply. For simplicity, assume firms 1 and 2 offer the same loyalty discount d, and have respectively signed up a θ_1 and θ_2 share of buyers, where $0 < \theta_1 + \theta_2 < 1$. Call θ_1 whichever is larger, so that $\theta_1 > \theta_2$.

For any uncommitted price, P_2, that the firm 2 chooses, firm 1 has two options. First, it can deprive firm 2 of all uncommitted sales by lowering firm 1's uncommitted price to $P_2 - \varepsilon$, thus selling at $P_2 - \varepsilon$ to a $1-\theta_1-\theta_2$ share of buyers, and at $P_2 - \varepsilon - d$ to a θ_1 share of buyers. Second, it can concede all uncommitted buyers, but still make all sales to the committed buyers, in which case it will maximize profits by charging $P_1 - d = P_m$ to a θ_1 share of buyers.

The second option will be more profitable to firm 1 if:

$$\theta_1 \cdot (P_m - \boldsymbol{C})(A - P_m) > (1-\theta_1-\theta_2)(P_2 - \boldsymbol{C})(A - P_2) + \theta_1 \cdot (P_2 - d - \boldsymbol{C})(A - P_2 + d).$$

This can be rearranged as:

$$(1-\theta_2)P_2^2 - 2[(1-\theta_2)P_m + \theta_1 d]P_2 + (1-\theta_2)\boldsymbol{C}(2P_m - \boldsymbol{C}) + [2dP_m + d^2 + (P_m - \boldsymbol{C})^2]\theta_1 > 0.$$

The Appendix proves this will be true as long as firm 2 charges no more than

$$P_2^* = P_m + \theta_1 d/(1-\theta_2) - [(1-\theta_1-\theta_2)((P_m - \boldsymbol{C})^2/(1-\theta_2) - \theta_1 d^2/(1-\theta_2)^2)]^{1/2}$$

and that P_2^* is always above P_r^* and more profitable for firm 2 than any alternative price as long as $P_2^* < P_m + d\theta_2/(1-\theta_1)$. If $P_2^* \geq P_m + d\theta_2/(1-\theta_1)$, the Appendix proves that firm 2 will find it more profitable to charge uncommitted buyers $P_m + d\theta_2/(1-\theta_1)$, and to charge its committed buyers $P_m - d \cdot (1-\theta_1-\theta_2)/(1-\theta_1)$, for an average price of P_m. This proves Proposition 5.

[13] I omit the math to determine the precise price under this scenario because it takes up too much space given the complexity of the resulting formulas.

Proposition 5. Suppose there are no economies of scale or both firms achieve them, and two firms engage in Bertrand competition and offer discounts for loyalty commitments, with firm 1 getting the larger foreclosure share, $\theta_1 > \theta_2$. Define $P_2^* = P_m + \theta_1 d/(1-\theta_2) - [(1-\theta_1-\theta_2)((P_m-C)^2/(1-\theta_2) - \theta_1 d^2/(1-\theta_2)^2)]^{1/2}$.

a. If $P_2^* \geq P_m + d\theta_2/(1-\theta_1)$, then firm 2 will sell to all uncommitted buyers at $P_m + d\theta_2/(1-\theta_1)$ and sell to all its committed buyers at $P_m - d\cdot(1-\theta_1-\theta_2)/(1-\theta_1)$, for an average price of P_m. Firm 1 will sell to all its committed buyers at P_m.

b. If $P_2^* < P_m + d\theta_2/(1-\theta_1)$, then firm 2 will sell to all uncommitted buyers at P_2^*, and sell to all its committed buyers at $P_2^* - d$. Firm 1 will sell to all its committed buyers at P_m.

The prices firm 2 charges to uncommitted buyers will always exceed P_r^*, the price it would have charged uncommitted buyers if only firm 1 had loyalty commitments. All the prices will exceed the but-for competitive level, C, which would have prevailed if neither offered loyalty discounts.

Because firms 1 and 2 are both offering loyalty commitments, buyers are better off picking a loyalty commitment from either firm 1 or firm 2 than remaining uncommitted. Thus, one would expect all buyers to accept a commitment from one of the firms, until there are no uncommitted buyers. When this equilibrium is reached, both firm 1 and firm 2 will charge their committed buyers P_m with a nominal list price of $P_m + d$ that no buyer pays.

Now suppose there is a third firm, firm 3, deciding whether to invest to become a serious rival. Firm 3 faces precisely the same situation as the rival faced in Section II, only with a cumulative foreclosure share that exceeds the single firm foreclosure share because $\theta_1 + \theta_2 > \theta_1$. This higher foreclosure share makes it more likely that the rival cannot achieve its minimum efficient scale. Further, if the rival does enter and achieve minimum efficient scale, the higher foreclosure share raises the rival's prices, because all the price formulas increase with increasing total θ.

VI. Implications

The analysis here disproves many commonly held beliefs about loyalty discounts. Most basically, many hold the misconception that loyalty discounts presumptively lower prices.[14] The above proves this is untrue – in every situation analyzed above, loyalty discounts raised prices above but-for levels. The word "discounts" deceptively suggests otherwise, but the nominal "discount" is just the difference between the uncommitted and committed prices that a firm chooses, and does not indicate prices lower than the levels that would have resulted without loyalty discounts. There is no sound economic reason to conflate real discounts from but-for levels with price differences conditioned on compliance with exclusionary terms. To the contrary, loyalty discounts perversely discourage discounting.

Several courts and scholars have claimed that loyalty discounts should be deemed presumptively or conclusively procompetitive if the discounted price is above cost,[15] or if the rival is pricing above its own costs.[16] The above disproves both these claims. In every situation, the discounted and rival prices are above cost, but the loyalty discount results in anticompetitive effects. Both claims miss the point that loyalty discounts discourage price-cutting by both users and nonusers, and cause prices to be above cost. The first claim also missed the point that loyalty commitments can raise rivals' costs above but-for levels.

More generally, many have argued that exclusionary conduct is not anticompetitive unless it involves a short-term profit sacrifice,[17] would not be profitable if it did not eliminate or impair rivals,[18] or does or could exclude an equally efficient rival.[19] The above disproves these claims. In all the above situations, the loyalty discounts have anticompetitive effects even though the conduct is always profitable, would remain profitable even without eliminating or impairing rivals, and whether or not the rival is equally efficient and stays in the market. Further, the equally efficient rival test misses the point that sometimes the loyalty discounts will create anticompetitive effects by making the rival less efficient.

---

[14] *See, e.g.,* Hovenkamp (2006).

[15] Hovenkamp (2006); Hovenkamp (2005), at 129, 132; Lambert (2005); *NicSand v. 3M*, 507 F.3d 442 (6th Cir. 2007); *Concord Boat v. Brunswick Corp.*, 207 F.3d 1039 (8th Cir. 2000). Others have rejected this claim, but without rigorous economic proof. *FTC v. Brown Shoe*, 384 U.S. 316 (1966); *LePage's v. 3M*, 324 F.3d 141 (3d Cir. 2003) (en banc).

[16] Lambert (2005); *NicSand.*

[17] Bork (1978); Posner (2001); Ordover & Willig (1981); Patterson (2003).

[18] Melamed (2006); Werden (2006).

[19] Posner (2001); Hovenkamp (2005), at 129, 132; Lambert (2005); Lave (2005); DG Competition (2005); *Cascade Health Solutions v. Peacehealth*, 502 F.3d 895 (9th Cir. 2007).

Another common general claim is that exclusionary agreements cannot be anticompetitive if buyers voluntarily agree to them.[20] Again, the above analysis disproves this claim. In all the situations, the buyers voluntarily agree to the loyalty discounts because doing so makes each individually better off, even though collectively they would be better off if none of them accepted. Relatedly, some have argued that loyalty discounts are anticompetitive only when they create a form of intraproduct bundling, by bundling each buyer's contestable demand for a product with its incontestable demand, such as when the buyer is a dealer with two sets of downstream buyers.[21] The above again proves this is untrue because none of the models assumed buyers had such divergent demands for the product of the firm using loyalty discounts.

Others more modestly assume that loyalty discounts cannot be anticompetitive unless they create a large enough foreclosure to impair rival efficiency.[22] The above proves that even this claim is untrue, because it turns out to miss the fact that loyalty discounts discourage discounting even if they do not affect rival efficiency at all. Relatedly, courts or scholars often say that exclusionary agreements should not be deemed anticompetitive unless they foreclose a substantial share of the market, with 20-40% often stated to be the level necessary to be "substantial" under U.S. antitrust law.[23] However, because the anticompetitive effects of loyalty discounts do not depend on the rival losing economies of scale, they persist even at low foreclosure levels. For example, suppose the foreclosure share were only 10%, with $P_m = 100$, $\boldsymbol{C} = 20$, and $d = 20$. Then Proposition 1 shows that the loyalty commitments would still cause user prices that are 400% over but-for levels and a rival price of $P_r = 26.3$ that is 31.5% above the but-for level. Increasing the foreclosure level does increase the anticompetitive effect, but even a relatively low foreclosure share can elevate prices substantially above but-for levels. This seems to support the position in EC competition caselaw, as well some U.S. cases, which have found loyalty discounts by

---

[20] Posner (2001); Bork (1978); *Concord Boat.* Others have rejected this claim. *Standard Fashion*; *Brown Shoe*.

[21] DG Competition (2005); *Canada v. Canada Pipe*, 2005 Canada Comp. Trib. 3; Hovenkamp (2005), at 129-130.

[22] Jacobson (2002); Brodley & Ma (1993).

[23] Wright (2006); *Twin City Sportservice v. Charles O. Finley & Co.*, 676 F.2d 1291 (9th Cir. 1982) (24% sufficed); *Stop & Shop Supermarket v. Blue Cross*, 373 F.3d 57 (1st Cir. 2004) (30–40%); *Concord Boat* (must be substantial); Areeda (1991) (20% presumptively unreasonable); Hovenkamp (1998) (20% with HHI over 1800 presumptively unreasonable).

firms with market power illegal without proof of a substantial foreclosure share, in cases where the loyalty conditions lacked any efficiency justification.[24]

At least one court has suggested that loyalty discounts cannot be anticompetitive if the discount levels are low, such as 1-3%.[25] However, while the above proves that increasing the discount level does increase rival prices further above but-for levels, it also proves that even small discount levels can elevate prices substantially. Indeed, for loyalty commitments, even a discount level of 0% (that is a promise to treat loyal buyers no worse than nonloyal buyers) leads to anticompetitive effects. For example, suppose the foreclosure share is 50%, $P_m = 100$, $\boldsymbol{C} = 20$, and the discount level is zero. Then, Proposition 1 shows that the loyalty commitment would still cause the user to price 400% above the competitive level, and cause the rival to charge $P_r = 43.4$, or 117% above but-for levels. However, a zero discount would not cause any effect without commitment, because then Proposition 3 shows the rival price would equal $\boldsymbol{C}$. However, even without commitment, a loyalty discount of 1-3% would raise rival and user prices to 21-23, which is 5-15% above but-for levels, more than significant given the 5% standard of significance used in the U.S. merger guidelines.[26]

Another issue of lively debate is whether exclusionary agreements should be deemed presumptively or conclusively procompetitive if they are terminable or require no commitment, with many courts and scholars asserting the answer is yes.[27] The above disproves that claim. Indeed, Proposition 3 proves that, even when loyalty discount agreements require no commitment at all, they can raise prices greatly above but-for levels. For example, if $P_m = 100$, $\theta = .5$, $\boldsymbol{C} = 20$, and $d = 20$, then a loyalty discount

---

[24] Case 85–76, *Hoffmann–La Roche v. Commission*, 1979 E.C.R. 461; *Brown Shoe; Le Page's;* Microsoft v. U.S., 253 F.3d 34, 68 (D.C. Cir. 2001) (en banc).

[25] *Concord Boat.*

[26] U.S. Department of Justice & Federal Trade Commission, *Horizontal Merger Guidelines*, §1.11 (1992, revised 1997).

[27] Hovenkamp (1998); Hovenkamp (2005), at 129; Wright (2006); *Concord Boat.*; CDC Technologies., Inc. v. IDEXX Labs, Inc., 186 F.3d 74 (2d Cir. 1999); Omega Envtl. v. Gilbarco, Inc., 127 F.3d 1157 (9th Cir. 1997); Thompson Everett, Inc. v. Nat'l Cable Adver., 57 F.3d 1317 (4th Cir. 1995); U.S. Healthcare v. Healthsource, 986 F.2d 589, 596 (1st Cir. 1993); Roland Mach. v. Dresser Indus., 749 F.2d 380 (7th Cir. 1984). Others have rejected this claim. *Brown Shoe* (condemning agreement terminable at will); Standard Oil v. United States, 337 U.S. 293 (1949) (condemning agreement terminable upon thirty days notice); Standard Fashion v. Magrane–Houston Co., 258 U.S. 346 (1922) (condemning agreement terminable upon three months notice); United States v. Dentsply, Intl., 399 F.3d 181 (3d Cir. 2005); *LePage's*.

without any commitment would cause the user and rival to both price at 40, which is 100% above the but-for level. For any given discount and foreclosure level, loyalty discounts without commitments result in somewhat lower prices than those with commitments. However, the resulting prices are still above cost, and the anticompetitive result is more stable because buyers who agree to the loyalty discounts never do any worse than those who do not.

Others have stated that loyalty discounts cannot be anticompetitive if they require significantly less than 100% exclusivity.[28] The above shows this is false. A threshold lower than 100% does not at all alter the anticompetitive effects of loyalty discounts without commitments. While a lower threshold reduces the anticompetitive effects for loyalty discounts with commitments, they remain significant and at least as high as the anticompetitive effects of loyalty discounts without commitments. Relatedly, some have suggested that if buyers buy more from the user than the sub-100% threshold required by their loyalty discount, then it is unlikely to be anticompetitive.[29] The above again disproves this. Indeed, for loyalty discounts without commitments, anticompetitive effects result though buyers always buy more that the threshold from the user. For loyalty discounts with commitments, they result even though buyers often make above-threshold purchases from the user.

Some have argued that loyalty discounts cannot create any anticompetitive effects if other firms can also use them.[30] Proposition 5 proves, to the contrary, that the anticompetitive effects are exacerbated if multiple firms use loyalty discounts. Proposition 5 also bears on the appropriateness of using a cumulative foreclosure approach that aggregates the foreclosure shares produced by multiple sellers. Although U.S. Supreme Court cases and EC guidelines have long used a cumulative foreclosure approach,[31] some have argued cumulative foreclosure has no economic basis.[32] The above disproves this argument. Although anticompetitive effects persist at low foreclosure levels, the cumulative effect of foreclosure by two firms is to raise prices above the levels that would have been created by the foreclosure of only one

[28] *Concord Boat.* Others have rejected this claim. *Brown Shoe* (75% threshold sufficed); *Microsoft* (D.C. Cir. 2001) (en banc)(75% threshold sufficed).

[29] *Concord Boat.*

[30] *NicSand*.

[31] FTC v. Motion Picture Advertising Service, 344 U.S. 392 (1953); *Standard Oil*; EC Guidelines on Vertical Restraints §2.1, O.J. 2000, C291/1.

[32] Hovenkamp (2004); Paddock Publ'ns. v. Chicago Tribune, 103 F.3d 42 (7th Cir. 1996).

of the firms. Further, the effect is to make loyalty discounts more stable by driving buyers into commitments with one of the firms offering them and deterring production by other firms. Thus, if foreclosure levels are used to screen out cases based on the likely size of anticompetitive effects, then it makes more sense to look at cumulative foreclosure than single firm foreclosure.

Any of the above anticompetitive effects might be offset by efficiencies. Such efficiencies were implicitly excluded from my model, and the models used in prior articles, because the models all assume cost and demand curves that are not altered by the existence of loyalty discounts. This precludes the possibilities that loyalty discounts might lower production costs or increase product value. In other words, it assumes exclusion "naked" of any efficiency justifications. If loyalty discounts can be demonstrated to have efficiencies that cannot be advanced by less restrictive alternatives, such as volume-based discounts, then the net effects of loyalty discounts might increase net efficiency, lower prices, or otherwise benefit consumers despite some anticompetitive effects. This article proves the effects of loyalty discounts only on the assumption that they are not necessary to achieve efficiencies. However, lower prices are not themselves an efficiency justification for loyalty discounts, as some have thought,[33] both because firms can lower prices without conditioning those prices on loyalty, and because this article proves that, absent some productive efficiency, conditioning price reductions on such loyalty conditions tends to raise, not lower, prices.

On the other hand, the anticompetitive effects predicted above are understated because of the extreme assumption of Bertrand competition, especially as to anticompetitive effects on rivals. If we made more realistic assumptions of imperfect competition, loyalty discounts would be more likely to both create adverse effects on rival competitiveness and lead to anticompetive equilibria. The former would be true if, for example, one more realistically assumed that switching costs exist, that supply elasticity is limited so that output cannot instantly be expanded, or that differentiated demand meant that loyalty discounts bundled contestable with incontestable demand. The latter would be true if imperfect competition meant that even two firms operating at efficient scale would produce above-cost prices for free buyers from which loyalty discounts could be offered, making it even easier to arrive at equilibria in which buyers agree to anticompetitive loyalty commitments and do not breach or terminate

[33] *Concord Boat.*

them. Such above-cost pricing would result if we assumed that firms either operate on a differentiated market, view competition as a multi-period game with no fixed endpoint (and thus coordinate on uncommitted prices), or that expanding output requires advance planning so that firms pick outputs rather than price (and thus engage in Cournot or Stackelberg competition).

REFERENCES

Areeda, Philip. 1991. *Antitrust Law*, Vol. IX: 375, 377, 387.

Bork, Robert H. 1978. *The Antitrust Paradox: A Policy at War with Itself,* 306-309. New York: Basic Books.

Brodley, Joseph F. & Ching-to Albert Ma. 1993. "Contract Penalties, Monopolizing Strategies, and Antitrust Policy," *Stanford Law Review* 45: 1161-1213.

DG Competition. 2005. "Discussion Paper on the Application of Article 82 of the Treaty to Exclusionary Abuses."

Elhauge, Einer. 2003. "Defining Better Monopolization Standards." *Stanford Law Review* 56*:* 253-344.

Farnsworth, E. Allan. 2004. *Contracts* (4th ed.). New York: Aspen Publishers.

Hovenkamp, Herbert. 1998. *Antitrust Law* Vol.. XI, at 152, 160, 167-69.

Hovenkamp, Herbert. 2004. "Group Purchasing Organization (GPO) Purchasing Agreements and Antitrust."

Hovenkamp, Herbert. 2005. *Antitrust Law* Vol.. XI (2d ed.).

Hovenkamp, Herbert. 2006. "Discounts and Exclusion." *Utah Law Review,* 2006: 841-861.

Jacobson, Jonathan M. 2002. "Exclusive Dealing, 'Foreclosure,' and Consumer Harm," *Antitrust Law Journal,* 70: 311-369.

Lambert, Thomas A.. 2005. "Evaluating Bundled Discounts." *Minnesota Law Review,* 89: 1688-1757.

Landeo, Claudia and Kathryn Spier. 2007. "Naked Exclusion: An Experimental Study of Contracts with Externalities." Harvard Law & Economics Discussion Paper No. 604 (November 22, 2007).

Lave, Jonathan M. 2005. "The Law and Economics of De Facto Exclusive Dealing." *The Antitrust Bulletin,* 50: 143-180.

Meese, Alan J. 2005. “Exclusive Dealing, the Theory of the Firm, and Raising Rivals’ Costs,” *The Antitrust Bulletin*, 50: 371-439.

Melamed, Douglas A. 2006. “Exclusive Dealing Agreements and Other Exclusionary Conduct--Are There Unifying Principles?” *Antitrust Law Journal* 73: 375-412.

Ordover, Janusz A. & Robert D. Willig. 1981. “An Economic Definition of Predation.” *Yale Law Journal* 91: 8-52.

Mark R. Patterson. 2003. “The Sacrifice of Profits in Non-Price Predation,” *Antitrust*, at 37.

Posner, Richard A. 2001. *Antitrust Law: An Economic Perspective.* Chicago: University of Chicago Press (2d ed.) .

Rasmusen, Eric B., J. Mark Ramseyer, J., and John S, Wiley. 1991. "Naked Exclusion", *American Economic Review,* 81(5):1137–45.

Schwartz, Alan and Robert E. Scott. 2003. "Contract Theory and the Limits of Contract Law." *Yale Law Journal* 113: 541- (2003)

Segal, Ilya R., and Michael Whinston. 2000. “Naked Exclusion: Comment.” *American Economic Review,* 90: 296-309.

Simpson, John and Abraham L. Wickelgren. 2007. “Naked Exclusion, Efficient Breach, and Downstream Competition.” *American Economic Review,* 97: 1305-1320.

Tirole, Jean. 1988. *The Theory of Industrial Organization.* Cambridge, Massachusetts: MIT Press.

Werden, Gregory J. 2006. “Identifying Exclusionary Conduct Under Section 2: The ‘No Economic Sense’ Test, *Antitrust Law Journal* 73: 413-433.

Wright, Joshua D. 2006. “Antitrust Law and Competition for Distribution.” *Yale Journal on Regulation* 23: 169-207.

APPENDIX

Proof of Proposition 1. As the paper showed, the user will not undercut the rival price to uncommitted buyers as long as

$$P_r^2 - 2(P_m+\theta d)P_r + 2\boldsymbol{C}P_m - \boldsymbol{C}^2 + 2\theta dP_m + \theta d^2 + \theta \cdot (P_m-\boldsymbol{C})^2 > 0.$$

Because this form repeats throughout this article, it is worth pointing out that for any inequality $ax^2 - 2bx + c > 0$, where a and b are both positive, taking the first and second derivative will show there is a minimum at $x = b/a$. The value of the left hand formula at the minimum will be $-b^2/a + c$. Thus, if $c > b^2/a$, the inequality will always be satisfied. If $c < b^2/a$, the formula is negative at its minimum and will become positive (and thus satisfy the inequality) only if x is below the lower quadratic root or above the higher root, which are $b/a \pm [(b/a)^2 - c/a]^{1/2}$.

Thus, the above inequality is always satisfied if

$$2\boldsymbol{C}P_m - \boldsymbol{C}^2 + 2\theta dP_m + \theta d^2 + \theta \cdot (P_m-\boldsymbol{C})^2 > (P_m+\theta d)^2$$

which can be rearranged as $d\theta^{1/2} > P_m - \boldsymbol{C}$. When that is the case, the rival can charge any price to uncommitted buyers without causing the user to try to undercut it, and thus the rival will pick the profit-maximizing price of P_m

If $d\theta^{1/2} < P_m - \boldsymbol{C}$, then using the above and simplifying, the inequality will be satisfied only if P_r is above or below the respective quadratic roots, which we can simplify as:

$$P_m + \theta d \pm [(1-\theta)((P_m-\boldsymbol{C})^2 - \theta d^2)]^{1/2}$$

Because the midpoint of the two roots is higher than P_m, then at the higher root P_r must be further away from P_m, which we can show is always less profitable. To see why assume any set of possible prices $P_m + X \pm Y$, where X and Y are both positive. Then the lower solution will earn more than the higher solution if

$$(P_m + X - Y - \boldsymbol{C})(A - P_m - X + Y) > (P_m + X + Y - \boldsymbol{C})(A - P_m - X - Y)$$

which because $A = 2P_m - C$, can be simplified to being true whenever

$$XY > 0,$$

which is always true because X and Y are both positive. Thus, the rival will always choose the lower solution over any price at or above the higher solution.

The rival will thus charge a price up to

$$P_r^* = P_m + \theta d - [(1-\theta)((P_m-\boldsymbol{C})^2 - \theta d^2)]^{1/2}$$

and be able to sell to all the uncommitted buyers without inducing the user to undercut its price. If $P_r^* \geq P_m$, then the rival will charge P_m since that price will earn more profits from uncommitted buyers than a higher price. P_r^* will be $\geq P_m$ only if

$$\theta d \geq [(1-\theta)((P_m-\boldsymbol{C})^2 - \theta d^2)]^{1/2},$$

which simplifies to being true only if

$$P_m\text{-}\boldsymbol{C} \leq [\theta/(1\text{-}\theta)]^{½}d.$$

If $P_r^* < P_m$, then the rival will charge P_r^* because any lower price earns less profit. $P_r^* > \boldsymbol{C}$ if $P_m + \theta d - [(1\text{-}\theta)((P_m\text{-}\boldsymbol{C})^2 - \theta d^2)]^{½} > \boldsymbol{C}$, which simplifies to,

$$\theta(P_m\text{-}\boldsymbol{C})^2 + 2(P_m\text{-}\boldsymbol{C})\theta d + \theta d^2 > 0.$$

Since $P_m > \boldsymbol{C}$, this is true whenever $\theta > 0$, that is whenever any buyer accepts the loyalty discount. Note that d need not be > 0.

Thus, as long as any buyer accepts the loyalty discount, there is always a rival price $P_r^* > \boldsymbol{C}$ that the rival can charge that will cause the user to keep the price to committed buyers equal to $P_{m,}$ with the user thus offering uncommitted buyers $P_m + d$ but being undercut by P_r^*, so that the rival makes all sales to uncommitted buyers at P_r^* and the user makes all sales to committed buyers at P_m.

Proof of Lemma 1. In the posited case, the gain to the user from the agreement of each buyer will be $(P_m\text{-}\boldsymbol{C})(Q_m/N)$, which since $Q_m = P_m\text{-}\boldsymbol{C}$ is the same as $(1/N)(P_m\text{-}\boldsymbol{C})^2$. The loss to each agreeing buyer will be the difference between their consumer surpluses at P_r^* and P_m, which will be $(1/2)(A\text{-}P_r^*)Q_r/\mathrm{N}$ - $(1/2)(A\text{-}P_m)Q_m/N$, which is the same as $(1/2N)(2P_m\text{-}\boldsymbol{C}\text{-}P_r^*)^2$ - $(1/2N)(P_m\text{-}\boldsymbol{C})^2$. Thus, the user can offer a sidepayment that induces buyers with this expectation to agree as long as

$$(1/N)(P_m\text{-}\boldsymbol{C})^2 > (1/2N)(2P_m\text{-}\boldsymbol{C}\text{-}P_r^*)^2 - (1/2N)(P_m\text{-}\boldsymbol{C})^2,$$

which can be expressed as

$$P_r^{*2} - 2(2P_m\text{-}\boldsymbol{C})P_r^* - 2\boldsymbol{C}^2 + 2P_m\boldsymbol{C} + P_m^{\ 2} < 0.$$

The left hand formula has a minimum at $P_r^* = 2P_m\text{-}\boldsymbol{C}$, at which it takes the value $-3(P_m\text{-}\boldsymbol{C})^2$, which is always negative. It stays negative (and thus satisfies the inequality) as long as P_r^* is between the roots $2P_m\text{-}\boldsymbol{C} \pm (3)^{.5}(P_m\text{-}\boldsymbol{C})$. We can ignore the upper bound because no rival would not offer such a price, given that doing it is greater than P_m. Thus, a profitable sidepayment can be made as long as buyers expect $P_r^* \geq 2P_m\text{-}\boldsymbol{C} - (3)^{.5}(P_m\text{-}\boldsymbol{C})$, which with rounding can be simplified as

$$P_r^* \geq .27P_m + 0.73\boldsymbol{C}.$$

Proof of Proposition 3. As the paper showed, the user will not undercut the rival price to unconditioned buyers as long as

$$(1\text{-}\theta)P_r^{\ 2} - 2[(1\text{-}\theta)P_m + \theta d]P_r + (1\text{-}\theta)2\boldsymbol{C}P_m - (1\text{-}\theta)\boldsymbol{C}^2 + 2\theta dP_m + \theta d^2 > 0$$

This inequality is always satisfied if $P_m\text{-}\boldsymbol{C} < [\theta\text{-}2\theta^2]^{1/2}d/(1\text{-}\theta)$, in which case the rival can pick any price without being undercut on unconditioned buyers, so it will price at the profit-maximizing price, P_m.

If P_m-$\boldsymbol{C} > [\theta\text{-}2\theta^2]^{1/2}d/(1\text{-}\theta)$, then the inequality will be satisfied only if P_r is above or below the respective quadratic roots, which we can simplify as:

$$P_r = P_m + d\theta/(1\text{-}\theta) \pm [(P_m\text{-}\boldsymbol{C})^2 + \theta d^2(2\theta\text{-}1)/(1\text{-}\theta)^2]^{½}.$$

The higher of the two solutions is above P_m and thus we know that any price above if will be even further away from the profit-maximizing price and thus earn less profits. Further, because the higher solution is further away from P_m, we know it is less profitable than the lower solution, given the proof above. Thus, the rival will always choose the lower solution over any price at or above the higher solution.

The rival will thus charge a price up to

$$P_r^{**} = P_m + d\theta/(1\text{-}\theta) - [(P_m\text{-}\boldsymbol{C})^2 + \theta d^2(2\theta\text{-}1)/(1\text{-}\theta)^2]^{½}.$$

and be able to sell to all the unconditioned buyers without triggering a price cut that undercuts its price. If $P_r^{**} \geq P_m$, then the rival will charge P_m since that will earn more profits from unconditioned buyers than a higher price. P_r^{**} will be $\geq P_m$ only if

$$d\theta/(1\text{-}\theta) \geq [(P_m\text{-}\boldsymbol{C})^2 + \theta d^2(2\theta\text{-}1)/(1\text{-}\theta)^2]^{½}.$$

which simplifies to being true only if

$$P_m\text{-}\boldsymbol{C} \leq [\theta/(1\text{-}\theta)]^{½}d.$$

If $P_r^{**} < P_m$, then the rival will charge P_r^{**} as long as $P_r^{**} > \boldsymbol{C}$. This condition will be met whenever

$$P_m + d\theta/(1\text{-}\theta) - [(P_m\text{-}\boldsymbol{C})^2 + \theta d^2(2\theta\text{-}1)/(1\text{-}\theta)^2]^{½} > \boldsymbol{C},$$

which can be rearranged as when

$$2(P_m\text{-}\boldsymbol{C}) > d(\theta\text{-}1)/(1\text{-}\theta)$$

Because $\theta \leq 1$, this is always true. Thus, there always exists a $P_r^{**} > \boldsymbol{C}$ that the rival can charge that will cause the user to sell to conditioned buyers at $P_r^{**} - \varepsilon$, while the rival sells to unconditioned buyers at P_r^{**}.

Proof of Proposition 5. As the paper showed, firm 1 will not undercut firm 2's price to uncommitted buyers as long as

$$(1\text{-}\theta_2)P_2^{\,2} - 2[(1\text{-}\theta_2)P_m + \theta_1 d]P_2 + (1\text{-}\theta_2)\boldsymbol{C}(2P_m\text{-}\boldsymbol{C}) + [2dP_m+d^2+(P_m\text{-}\boldsymbol{C})^2]\theta_1 > 0$$

This inequality is always satisfied if $d[\theta_1/(1\text{-}\theta_2)]^{1/2} > P_m - \boldsymbol{C}$. When that is the case, firm 2 can charge any price to uncommitted buyers without causing firm 1 to try to undercut it. Firm 2 will thus pick the price that maximizes its profits for both the combination of its committed buyers and these uncommitted buyers. The tradeoff is the same as that faced by an user without any rival if we adjust for the different ratios of uncommitted to committed buyers. Thus, given Proposition 2a, firm 2 will charge uncommitted buyers $P_m + d\theta_2/(1\text{-}\theta_1)$ and charge committed buyers $P_m - d\cdot(1\text{-}\theta_1\text{-}\theta_2)/(1\text{-}$

θ_1), for an average price of P_m.

If $d[\theta_1/(1-\theta_2)]^{1/2} < P_m$ - $\boldsymbol{C}$, then the inequality will be satisfied only if P_r is above or below the respective quadratic roots, which we can simplify as:

$$P_m + \theta_1 d/(1-\theta_2) \pm [(1-\theta_1-\theta_2)((P_m-\boldsymbol{C})^2/(1-\theta_2) - \theta_1 d^2/(1-\theta_2)^2)]^{1/2}$$

The solutions above the upper root can be rejected for reasons noted in prior proofs. Thus, firm 2 can charge up to

$$P_2^* = P_m + \theta_1 d/(1-\theta_2) - [(1-\theta_1-\theta_2)((P_m-\boldsymbol{C})^2/(1-\theta_2) - \theta_1 d^2/(1-\theta_2)^2)]^{1/2}$$

and be able to sell to all the uncommitted buyers without firm 1 undercutting its price to uncommitted buyer. Because $\theta_1 > \theta_2$, and this price is the price at which firm 1 just breaks even between selling to the uncommitted buyers and selling to θ_1 buyers at the monopoly price, then it must be the case that this price is more profitable to firm 2 than forgoing the uncommitted buyers and selling to θ_2 buyers at $P_{m.}$

If $P_2^* > P_m + d\theta_2/(1-\theta_1)$, then firm 2 will charge $P_m + d\theta_2/(1-\theta_1)$ to uncommitted buyers since that price will earn firm 2 more profits than a higher price. This will be the case only if $P_2^* > P_m + d\theta_2/(1-\theta_1)$, which unfortunately does not simplify nicely. Otherwise, firm 2 will charge P_2^* to uncommitted buyers. Because every term in P_2^* makes it larger than P_r^*, it must be true that $P_2^* > P_r^*$.