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Settling Lawsuits with Pirates*

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Abstract

A firm licenses a product to overlapping generations of heterogeneous consumers. Consumers may purchase the product, pirate/steal it, or forego it. Higher consumer types enjoy higher gross benefits and are caught stealing at a higher rate. The firm may commit to an out-of-court settlement policy that is "soft" on pirates, so high-types purchase the product and low-types steal it until caught. Settlement contracts, which include both cash payments and licenses for future product use, facilitate price discrimination. Settlement may either create social value by expanding the market or destroy value by limiting market access and possibly deterring more efficient entrants.

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As long as they're going to steal it, we want them [consumers] to steal ours ... then we'll somehow figure out how to collect sometime in the next decade.[1] *(Bill Gates, 1998)*

1 Introduction

Piracy poses a major challenge for firms selling products protected by intellectual property rights (IPRs). According to recent estimates, 37% of software installed on personal computers is unlicensed and the global commercial value of pirated software is about $46 billion.[2] So-called "seed pirates," farmers who save and use patented seeds from one season to the next in violation of their contracts, have long vexed agricultural biotechology companies.[3] Counterfeit pharmaceutical products involve global sales of more than $150 billion each year.[4] Many firms have been taking legal action, namely litigation and settlement, to enforce their intellectual property rights. In 2013 alone, Microsoft settled more than three thousand piracy cases in more than 40 countries.[5] In the same year, Monsanto filed lawsuits against hundreds of farmers and small businesses, most of which were subsequently settled out of court.[6]

Out-of-court settlements between firms and pirates are often "bundled" with purchase agreements requiring the pirates to license the product in the future and/or to buy additional goods and services from the firms. For example, Microsoft settled a copyright infringement case with one unit of a leading security company in China who also agreed to buy Microsoft software worth about $480,000.[7] In another example, enterprise-software provider Apptricity sued the U.S. Army for the widespread unauthorized use of its products. Their settlement agreement specified a payment of $50 million for past infringement plus a license for ongoing use.[8] In 2008, Monsanto's settlement agreement with a large soybean farmers' co-operative

[1] Schlender, Buffett, and Gates (1998).

[2] From 2018 survey conducted by the Business Software Alliance (BSA). The piracy rate was 57% in Asia Pacific and Central and Eastern Europe, 26% in Western Europe, and 16% in North America. See https://www.bsa.org/files/2019-02/2018_BSA_GSS_Report_en.pdf.

[3] See https://www.centerforfoodsafety.org/reports/1770/seed-giants-vs-us-farmers

[4] See PwC report file:///C:/Users/kspier/AppData/Local/Temp/fighting-counterfeit-pharmaceuticals-1.pdf. The World Health Organization (WHO) estimates that counterfeits comprise the majority of drugs sold in some developing countries.

[5] https://news.microsoft.com/2013/07/09/microsoft-settles-3265-software-piracy-cases-in-us-and-abroad/. Similarly, Adobe has settled many lawsuits with unauthorized users. See https://www.cnet.com/news/adobe-settles-in-piracy-case/.

[6] See https://www.centerforfoodsafety.org/reports/1770/seed-giants-vs-us-farmers

[7] "Software Company Settles Piracy Suit with Citic Unit," *Wall Street Journal*, July 16, 2010. This case is not unique. Among 64 copyright infringement lawsuits filed by Microsoft with public court records during 2012-2014 in China, 25 cases were settled and some of the settlement agreements required defendants to purchase Microsoft products. (Source: Collected by the authors from various court records in China).

[8] *Apptricity Corporation vs. USA; 2013 Nat. Jury Verdict Review LEXIS 284*. See also "U.S. Army Pays

was bundled with a purchase agreement. The farmers agreed to buy \$1.1 million in seed products from Monsanto over the next six years.[9]

Settling lawsuits with pirates, and possibly bundling cash settlements for past offenses with licenses or purchase agreements for future use, can be a very profitable business strategy and provides another way for firms to monetize their IPRs. It has been recognized in the academic literature and in practice that accommodating pirates may help firms to penetrate new markets (especially given network effects and consumers' switching costs).[10] However, the role of ex-post IPR enforcement and the design of settlement contracts bundled with licenses have not been previously explored.

This paper considers a continuous-time model of an incumbent firm and overlapping generations of indefinitely-lived consumers. The firm makes a proprietary product or service at zero marginal cost. Consumers have the choice to purchase the product, pirate or steal it,[11] or forego its use. There are two consumer types, high and low, where high-type consumers (1) enjoy a higher gross benefit from using the product (purchased legitimately or pirated) and (2) face a higher rate of detection if they steal the product.[12] In our baseline model, we assume that the firm can commit to its settlement policy and that once a consumer is caught stealing they cannot steal again. We consider the legal environment with either strong or weak protections of property rights. In other words, the court-ordered damage awards can be high or low. We show that the firm can generate higher profits if it accommodates piracy and bundles settlement with a license for future use.

We begin by considering an environment with strong property rights where court-ordered damage awards are sufficiently high. Absent private settlement agreements, piracy is deterred. However, by committing to a private settlement policy that is "soft" on pirates (i.e. a low damage payment bundled with a license for future use), the firm can induce the high-type consumers to purchase the product ex ante and the low-type consumers to steal it and pay for the product ex post (when and if they are caught).[13] If the incentive

Out \$50m in Software Piracy Suit," *ETMAG.com*, Dec. 2, 2013.

[9]See https://www.patentdocs.org/2008/09/monsanto-announ.html. Visual-media company Getty Images sent countless settlement demand letters to users of unauthorized photographs. Some gave pirates the option to purchase photographs at less-than-market rates or even free of charge. See a sample letter at http://audio.hackerfactor.com/2014/getty-takedown-20140715.pdf.

[10]See the literature review. According to an interview with Bill Gates in *Fortune* in 2007: "[I]t's easier for our software to compete with Linux when there's piracy than when there's not."See https://finance.yahoo.com/news/yes-chinese-piracy-lost-microsoft-123104842.html.

[11]We use the words "pirate" and "steal" to refer to the unauthorized use of materials protected by IPR. In practice, enforcement actions are typically pursued privately by the IP owners rather than by public agencies. In the US, criminal prosecution for copyright infringement is generally reserved for commercial settings where counterfeit products are sold for commercial gain. See United States Code Volume 17, Sections 502-506.

[12]Consumers who use a piece of software more frequently, for example, may derive greater value from the product and face a higher detection rate. Additional discussion is provided in Section 2.

[13]Observers have noted "[the overall proprietary software industry's] anti-piracy crusade as

compatibility constraint for the high-type consumers is slack (so the high types are not tempted to steal), the firm will use long-term licenses in the settlement agreements with the low-type consumers, thereby capturing all of the ongoing consumer surplus. If the incentive compatibility constraint binds, then the firm will either shorten the license duration in the settlement contract (to make stealing less attractive for the high-type consumers) or reduce the purchase price (to make legitimate purchases more attractive for the high-type consumers).

Our model delivers normative insights as well. When property rights are strong, settlement with licensing may be good or bad for social welfare (compared to the no-settlement benchmark where pirates are taken to court). If the firm would charge a high price and exclude low-type consumers absent settlement, then the firm's "soft" settlement policy is good because the low-type consumers utilize the product until they are apprehended. If the firm would otherwise charge a low price and sell to both consumer types, however, then the firm's settlement policy may cause social welfare to fall. Social welfare falls if the low-type consumers are excluded from the market after they settle out-of-court and the license bundled with the settlement expires.

Next, we consider the environment with weak property rights where court-ordered damage awards are low. Absent settlement agreements, piracy cannot be fully deterred. Again, settlement with licensing could facilitate price discrimination and raise the firm's profits, except that the firm may allow both the high-type and low-type consumers to steal the product. In contrast to the environment where property rights are strong, out-of-court settlement with licensing (weakly) raises social welfare when property rights are weak. The environment with weak property rights is particularly relevant to piracy by individual users and small businesses as they are more likely to face financial constraints.

Finally, we extend the baseline model in several directions. First, we show that settlement agreements bundled with licensing are a valuable instrument for price discrimination even when the firm cannot commit to its settlement policy ex ante and all settlement negotiations take place in private ex post. Second, settlement with licensing has even greater strategic value when there are direct network externalities among consumers.[14] In particular, the firm settles with licenses of weakly longer duration to monetize the network benefits. Third, we illustrate how settlement with licensing may be employed by an incumbent firm to create a barrier to entry. When an incumbent firm faces a threat of entry, the firm extends the duration of the licenses in their settlement agreements. With longer purchase agreements or

a sophisticated dog-and-pony show... It has always been in Microsoft's interests for software to be available at two different prices." *New York Times*, Nov. 7, 2010. See https://www.nytimes.com/2010/11/07/technology/07piracy.html.

[14] Earlier literature on piracy emphasized network effects but did not explore civil litigation or settlement. See the literature review.

licenses, the rents that can be captured by potential entrants falls, thus creating a barrier to entry. We also show that settlement with licensing facilitates price discrimination when consumers who are apprehended for stealing once may steal again (recidivism) and under alternative technical assumptions.

Related Literature

Our paper is related to several strands of literature. First, our paper contributes to the literature on litigation and settlement.[15] In particular, the studies on patent litigation mostly focus on lawsuits against firms or sellers infringing intellectual property rights but not against users of pirated products. Settlement agreements in patent litigation may facilitate collusion among competitors and therefore reduce welfare (Mercer, 1989; Shapiro, 2006). Patent litigation can motivate patent holders to monitor and identify entrants (Crampes and Langinier, 2002), reveal valuable information to future entrants (Choi, 1998), and affect firms' investment incentives, which create endogenous disputes (Bessen and Mercer, 2006). As one exception, Choi and Gerlach (2018) analyze non-practicing entities' litigation strategies when they face multiple infringers (users).[16] Our paper diverges from this literature by investigating litigation against users who violate intellectual property rights and settlement bundled with licenses.

The literature on piracy addresses the question of how piracy or weak IPR enforcement affects firm profits. Novos and Waldman (1984), Johnson (1985), and Bae and Choi (2003) argue that piracy weakly reduces firm profits. Other studies, however, have identified possible channels for piracy or less stringent IPR enforcement to raise firm profits.[17] First, Conner and Rumelt (1991), Takeyama (1994), and Shy and Thisse (1999) argue that piracy can bring network effects and therefore allow firms to charge high prices for consumers who do not steal products. Casadesus-Masanell and Ghemawat (2006) further show that piracy creates network externalities and helps an incumbent to foreclose competitors. Second, Besen and Kirby (1989) and Bakos et al. (1999) argue that club-sharing among consumers reduces heterogeneity and therefore raises firm profits. Third, Duchene and Waelbroeck (2006) and Peitz and Waelbroeck (2006) argue that pirated products may disclose valuable information about product quality to potential consumers.[18] Finally, piracy may reduce price competition

[15]See Spier (2007), Daughety and Reinganum (2012), and Wickelgren (2013) for surveys.

[16]Luo and Mortimer (2017) conduct experiments to investigate patent holders' settlement strategies when facing infringers.

[17]Liebowitz (1985) provides empirical evidence that piracy may not harm publishers in the publishing industry.

[18]Luo and Mortimer (forthcoming) run experiments showing that copyright infringement can reveal product information to consumers or reduce their search costs.

between firms and increase their profits (Shy and Thisse, 1999; and Jain, 2008).[19] Buehler et. al. (2017) show in a static model that a sanction imposed on consumers evading payments could help firms to conduct price discrimination, though they do not consider settlement mechanisms bundled with licensing. Our paper contributes to this literature by allowing firms to monetize their property rights via civil settlement, and by exploring how settlement with licensing facilitates price discrimination and its welfare implications.[20]

Our paper is also related to the literature on contract duration and time considerations. Dybvig and Lutz (1993) characterize the profit-maximizing duration of warranties in a two-sided moral hazard framework. Guriev and Kvasov (2005) show that a well-designed contract duration (or termination notice period) solves hold-up problems in a bilateral relationship. Harstad (2012) and Battaglini and Harstad (2016) examine how the duration and expiration dates of environmental agreements affect participants' investment incentives. Guriev et al. (2015) show that the time structure of trade agreements is related to trade-facilitating investments. Green and Coq (2010) argue that the length of contracts with consumers affects the possibility of price collusion among competitors. In contrast, our paper examines how the duration of licenses that are bundled with settlement agreements can facilitate price discrimination and possibly create a barrier to entry.

Finally, the theoretical insights in this paper are relevant for policy makers and government agencies. Developing a deeper understanding of private IPR enforcement actions and out-of-court settlements is important for the design of regulations and competition policy. Allowing patent and copyright holders to include license agreements in their private settlement contracts facilitates price discrimination. This can create social value by increasing consumer use and expanding the market. However, settlement with licensing may also destroy social value by limiting market access and possibly deterring more efficient entrants.

Our paper is organized as follows. Section 2 sets up the baseline model. Section 3 first examines the no-settlement benchmark and then shows that settlement bundled with licenses for future use may be used by the firm as a price-discrimination mechanism. The social welfare implications of settlement are also explored. Section 4 discusses several relevant extensions, including limited commitment, network effects, entry deterrence, and recidivism. Section 5 offers concluding thoughts. The proofs of the main results are in Appendix A while the technical details for the extensions are in the online appendix.

[19] Peitz and Waelbroeck (2006) and Belleflamme and Peitz (2010) review the theoretical studies on piracy.

[20] Chen and Png (1999) considers how public enforcement of IPR affects software companies' selling prices and monitoring incentives. They do not consider civil litigation or private settlement between firms and stealing consumers.

2 The Model

The Firm and Consumers

A monopolist with zero marginal cost sells (or licenses) a product or service in continuous time to overlapping generations of consumers. There are two consumer types, $i \in \{L, H\}$, where λ_L and $\lambda_H = 1 - \lambda_L$ are the fractions of each type.[21] Consumers privately observe their types. Each active consumer exits the market at a constant rate $\alpha > 0$ and is instantly replaced by a new consumer of the same type. Thus, if a consumer is active at time $t = 0$, the time that the consumer exits the market, $\eta > 0$, is exponentially distributed with density $g(\eta) = \alpha e^{-\alpha\eta}$ and cumulative density $G(\eta) = 1 - e^{-\alpha\eta}$. The firm and the consumers discount time at a common rate $\rho > 0$.

The H-type consumers (i) enjoy higher gross benefits from product use and (ii) are detected at a higher rate if they steal. Specifically, we let v_i be the (instantaneous) gross valuation of a type-i consumer, and assume that $v_H > v_L$. A consumer enjoys the usage value v_i whether he or she purchases the product or steals it (thus pirated products are identical to purchased products).[22] Suppose a type-i consumer uses the product in every active period of his or her life. The consumer's discounted gross value is

$$V_i = \int_0^\infty \left(\int_0^\eta v_i e^{-\rho t} dt \right) \alpha e^{-\alpha\eta} d\eta = \frac{v_i}{\rho + \alpha}. \tag{1}$$

Letting $r = \rho + \alpha$ denote the (exit-adjusted) discount rate, we have

$$V_i = \frac{v_i}{r}. \tag{2}$$

Suppose that type-i consumers steal the product in every active period of their lives until they are detected by the firm. Detection is exogenous and follows a Poisson process where π_i is the rate of detection for type $i = L, H$.[23] We assume $\pi_H > \pi_L > 0$, so the H-types are detected at a higher rate than the L-types. In practice, more frequent users of stolen products derive higher values but are more likely to generate patterns of piracy that can be detected by the firm or its agents.[24] We assume that once a consumer is caught stealing,

[21] Having more consumer types would not affect the main insights regarding the use of settlement for price discrimination. If there were more consumer types, the types with very low values may not use the product at all.

[22] We abstract away from the possibility of product versioning. Interestingly, Athey and Stern (2015) find that software pirates tend to target the most advanced version of Microsoft Windows, suggesting that versioning might not succeed in capturing the price-sensitive users.

[23] In practice, firms can and do influence the probability of detection, for example, by changing the reward to whistleblowers, investing in technologies, and/or hiring more investigators. We have considered a simple extension with endogenous detection rates. Again, the firm would take a soft stand in detecting piracy, which facilitates price discrimination. The analysis is available upon request.

[24] According to Microsoft, they "identify activation patterns and characteristics that make

the consumer cannot steal again.[25] The time that a type-i consumer is caught stealing, τ, follows an exponential distribution $f_i(\tau) = \pi_i e^{-\pi_i \tau}$ with cumulative density $F_i(\tau) = 1 - e^{-\pi_i \tau}$. Notice that a type-$i$ consumer who begins stealing at time $t = 0$ will be caught on average at time $t = 1/\pi_i$.[26] Since v_i is the consumer's gross valuation, the fraction v_i/π_i is the type-i consumer's *average accumulated surplus* from stealing until caught (if the consumer does not exit the market).

To streamline the analysis, we make the following assumption.

Assumption 1: $\frac{v_H}{\pi_H} \leq \frac{v_L}{\pi_L}$.

Assumption 1 implies that the type-L consumer's *average accumulated surplus* from stealing is larger than the type-H consumer's. A sufficient condition is that the rate of detection is a weakly convex function of the consumer's gross valuation.[27] Note that we can treat the event that a stealing consumer is detected as statistically independent of the event that the consumer exits the market (a consumer may exit the market before getting caught).

We assume that if a consumer is detected stealing at time t, then the firm has hard proof that the consumer pirated the good at time t, so there are no "false positives." The firm does not directly observe a consumer's type, $i \in \{L, H\}$, the consumer's date of entry into the market (age or cohort), or the consumer's past history of misconduct. All of these things remain the consumer's private information. Note that consumers who are *not detected* at time t include consumers who hold valid licenses to use the product, consumers who are not using the product, and consumers who are pirating the product but remain undetected.

The Legal Environment

If a consumer is detected stealing then the firm can take the consumer to court and receive a civil damage award D. The parameter D, which we take as fixed, may be interpreted as the strength of the firm's intellectual property rights where higher values of D correspond to stronger IPR enforcement. The parameter D may also reflect the consumer's limited liability, as consumers sometimes lack the financial resources to pay damage awards in full.

it more likely than not that the IP address associated with the activation is an address through which pirated software is being activated." See http://ia800305.us.archive.org/10/items/gov.uscourts.wawd.214013/gov.uscourts.wawd.214013.1.0.pdf.

[25]In practice, once consumers are caught stealing, they are known to firms and enforcement authorities. Moreover, settlement contracts sometimes allow firms to conduct regular checks on consumers' places of business, so monitoring is heightened. See for example https://www.centerforfoodsafety.org/reports/1770/seed-giants-vs-us-farmers. See Section 4.4 for an extension with repeated stealing.

[26]This is the mean of the exponential distribution.

[27]For example, Assumption 1 holds if $\pi_i = \Pi(v_i)$ where $\Pi'(v_i) > 0$ and $\Pi''(v_i) \geq 0$. Section 4.5 analyzes the case where $\frac{v_H}{\pi_H} > \frac{v_L}{\pi_L}$, so the rate of detection is a concave function of the consumer's gross valuation. There, as here, settlement with licensing can be an effective way for the firm to price discriminate.

In practice, civil damages may be determined by statute, may reflect the firm's lost sales, or may reflect the consumer's unjust enrichment.[28] To receive statutory damages for copyright infringement, the firm need not provide any evidence of actual harm. In the US, statutory damages are in the \$750 to \$30,000 range for each pirated software program but may rise to \$150,000 for willful infringement.[29] Statutory damages can result in eye-popping damage awards. For example, in 2012, the court awarded Adobe Systems statutory damages of \$90,000 for each item that had retail value of \$150, resulting in a total award of \$2.52 million.[30] Perhaps because the level of statutory damages is so high (and the evidentiary requirements low) the majority of plaintiffs in copyright litigation seek statutory damages.[31]

Alternatively, damages may be based on estimates of the firm's lost profits and the consumer's illicit gains.[32] Recall that since the time of detection is exponentially distributed, a type-i consumer is caught after an average duration of $t = 1/\pi_i$. Thus, the court might benchmark against the market price and set damages equal to the average forgone profits from the period of infringement, $D = p/\pi_i$ where p is the flow price. Alternatively, the court might consider a hypothetical deal that would have been struck had the firm and consumer negotiated a personalized license ex ante. A damage award of $D = v_i/\pi_i$ represents the type-i consumer's average accumulated surplus (without interest).

The threshold $\overline{D} = v_L/\pi_L$ will play an important role in our equilibrium characterization. When property rights are strong in the sense that $D > \overline{D}$, then the L-types will get a negative expected payoff from stealing and paying D if caught. Moreover, since $v_H/\pi_H \leq v_L/\pi_L$ by Assumption 1, the H-types will get a negative expected payoff from stealing, too. If property rights are strong then the firm can deter all piracy by being "tough" on pirates.

Definition: Let $\overline{D} = v_L/\pi_L$. Property rights are *strong* if $D > \overline{D}$ and *weak* if $D \leq \overline{D}$.

Licensing and Settlement Policy

The firm commits to a policy or mechanism $\{P, X, T\}$ where P is the price of the product and X and T are the terms of the out-of-court settlement.[33] A consumer who chooses to

[28] In the US, "the owner of a copyright may collect either actual damages or statutory damages from an infringer." See U.S. Code Volume 17, Section 504.

[29] Multiple pirated copies of the same program would count as a single instance of infringement and statutory damages would not rise with the intensity of use. See U.S. Code Volume 17, Section 504(c). Courts in U.K. may also award punitive damages to right-holders based on Section 97(2) of the Copyright, Designs and Patents Act of 1988.

[30] *Feather v. Adobe Systems* 895 F. Supp. 2d 297 (D. Conn. 2012). Adobe sought the maximum of \$150,000 per item.

[31] Recent empirical work shows that 90% of plaintiffs in copyright disputes request statutory damages and 81% request enhanced damages. See Depoorter (2019).

[32] See Goldstein (2005).

[33] Our results remain the same under the more general mechanism with a menu of settlement contracts, $\{P, (P_1, X_1, T_1), (P_2, X_2, T_2)\}$, where P is the market price and P_1 (or P_2) is the (future) price of a perpetual license for any stealing consumer who chooses the settlement contract (X_1, T_1) (or (X_2, T_2)). The online

purchase the product pays a lump-sum amount P that gives the consumer the right to use the product in perpetuity.[34] In other words, P is the price of a long-term license. A consumer who chooses to steal the product uses the product free of charge and then, once detected, has the option to settle out of court. By paying X to the firm, the consumer avoids further prosecution and obtains a license of duration T. Note that through this mechanism, the firm is bundling the settlement of a lawsuit with a license (of possibly limited duration) for future use. Once the license expires, the consumer has the option (but not the obligation) to purchase a license at the market price P.[35]

Our assumption that the product is sold with a perpetual license is without loss of generality. Alternatively, the firm could sell short-term licenses or offer rental contracts with a limited duration T_0 at a lower price $P_0 = (1 - e^{-rT_0})P$. Because our environment is stationary, any consumer who chooses to purchase the limited-duration license would choose to renew the license upon expiration. Similarly, the firm could lease the product at an instantaneous price $p = rP$. By construction, a consumer is indifferent between paying P for a perpetual license, paying P_0 for a renewable license of duration T_0, and paying an instantaneous price $p = rP$ for an ongoing lease.

Our assumption that the settlement contract allows the pirate to use the product free of charge for a span of time T streamlines the notation but is not essential for our results. Equivalently, the settlement contract could include a requirement that the pirate pay a lump-sum fee of $(1 - e^{-rT})P$ for a license of duration T or, equivalently, an instantaneous price $p = rP$. Of course, if the pirate paid for the short-term license directly, the lump-sum cash settlement would be reduced from X to $X' = X - (1 - e^{-rT})P$.[36] Alternatively, the settlement contract could give the pirate an option to pay a lower instantaneous price $q < p$ for the duration of the license.[37]

Given the stationarity of the environment, the consumers' strategies simplify. Each active consumer will choose to (i) not consume the product at all, (ii) purchase a perpetual license for price P, or (iii) steal the product until caught and then exercise the option to settle the lawsuit or go to trial. Without loss of generality, we assume that consumers choose to

appendix shows that there exists a simple policy $\{P', X', T'\}$ that generates the same profits.

[34]In practice a firm may charge different prices based on observable information about consumers. We focus on residual information which is not observed by the firm but affects the probability for stealing consumers to be caught.

[35]Alternatively, one could assume that consumers who are caught stealing are prohibited from future purchases. This may further facilitate price discrimination by the firm. The analysis is available upon request.

[36]Formally, given any policy (P, X, T), there exists an equivalent mechanism where (1) the firm sells short-term licenses with duration T at the market price $(1 - e^{-rT})P$, and (2) the settlement contract requires the pirate to pay a cash settlement $X' = X - (1 - e^{-rT})P$ and buy one short-term licence of duration T at the market price.

[37]In this case, the lump-sum cash settlement could be reduced from X to $X' = X - (1 - e^{-rT})q/r$.

purchase the product whenever they are indifferent between purchasing and stealing it.[38]

The firm chooses its pricing and settlement policy, $\{P, X, T\}$, to maximize the present discounted value of the profit stream. For simplicity, we assume that, when given a choice between two revenue-equivalent policies, one where consumers purchase the product and another where consumers steal the product, the firm will choose the policy under which consumers buy the product. Similarly, given two revenue-equivalent policies, one leading to settlement and another leading to litigation, the firm will choose the policy leading to settlement.[39] Because the strategic environment is stationary, the policy that maximizes the firm's profit per consumer also maximizes its total profit.[40] For convenience, we shall focus on profits per consumer.

3 Settlement with Licensing

This section characterizes the equilibrium of the baseline model. We will show that the firm may commit to a settlement policy that is "soft" on pirates, so the L-types steal the product until caught. We will show that the settlement contracts, which involve cash payments and licenses for future use, facilitate price discrimination.

We begin by describing the consumer's decision whether to purchase the product at price P or steal it, and the consumer's subsequent decision to accept the settlement offer (X, T) or go to trial and pay D. A consumer who is caught stealing has the option to pay X to extinguish the firm's legal claim and enjoy the right to use the product for an additional duration T.

First, suppose a type-i consumer purchases the product. The consumer's net valuation is

$$B_i^1 = V_i - P. \tag{3}$$

Notice that if consumer types were observable, and if property rights were sufficiently strong, then the firm could price discriminate directly and charge each consumer type $P_i = V_i$.

Next, suppose that a type-i consumer steals the product until he is caught or exits the market. When the consumer is caught, the consumer will of course choose whichever option (trial or settlement) delivers greater surplus. If the consumer chooses trial, he pays D and

[38] Our results do not hinge on this assumption. Moreover, the assumption obviously holds if consumers face positive costs of litigation and settlement.

[39] These tie-breaking assumptions obviously hold if the firm faces positive (but negligible) costs of litigation and settlement. More generally, incorporating positive enforcement and litigation costs or consumers' potential loss of reputation would not change the main insights.

[40] Suppose that the firm's average profit per consumer is S, then the firm's total profit from the over-lapping generation is $R = S + \int_0^\infty (Re^{-\rho\tau})\alpha e^{-\alpha\tau} d\tau = S + \frac{\alpha}{\alpha+\rho} R$, which implies $R = \frac{\alpha+\rho}{\rho} S$.

purchases a license if $V_i \geq P$, giving the consumer a net surplus

$$-D + V_i - \min\{V_i, P\}. \tag{4}$$

If instead the consumer chooses settlement, he pays X, enjoys the product for an additional duration T, and then chooses whether to purchase a perpetual license at price P, giving the consumer a net surplus

$$-X + \int_0^T v_i e^{-rt} dt + e^{-rT} \max\{V_i - P, 0\}, \tag{5}$$

or equivalently,

$$-X + V_i - e^{-rT} \min\{V_i, P\}. \tag{6}$$

Since the consumer has the option to go to trial or accept the settlement offer, the consumer receives the maximum of (4) and (6). So, at the moment that they are caught stealing, the type-i consumer's net surplus is

$$V_i - \min\left\{D + \min\{V_i, P\}, X + e^{-rT} \min\{V_i, P\}\right\}. \tag{7}$$

We now construct the consumer's net valuation, B_i^2, from stealing the product. In our stationary environment, a consumer who chooses to steal the product at time $t = 0$ will continue to steal and enjoy the use of the product until they are caught or exit the market by natural attrition. Since detection is imperfect, the consumer discounts the possibility of future litigation and settlement. Using (7), one can show that a type-i consumer's net valuation from stealing is

$$B_i^2 = V_i - \theta_i \min\left\{D + \min\{V_i, P\}, X + e^{-rT} \min\{V_i, P\}\right\}, \tag{8}$$

where

$$\theta_i = \frac{\pi_i}{r + \pi_i} \tag{9}$$

is the discount factor that a type-i consumer places on being caught and sanctioned.[41] The proof of equation (8) is given in the appendix. One can easily verify that $\theta_H > \theta_L$ and $\frac{\theta_H}{\theta_L} < \frac{\pi_H}{\pi_L}$.[42]

Out-of-court settlement can be a highly effective way for the firm to extract V_i from

[41] When the rate of detection $\pi_i \to 0$, so the consumer is never caught stealing, then $\theta_i \to 0$. In this case, the type-i consumer's net present value from stealing is V_i (the consumer places zero weight on the sanctions). When the rate of detection $\pi_i \to \infty$, so a consumer who steals is caught immediately, then $\theta_i \to 1$.

[42] One can also verify that $\frac{\pi_H}{\pi_L} = \left(\frac{\theta_H}{1-\theta_H}\right)\left(\frac{1-\theta_L}{\theta_L}\right)$.

consumers. To illustrate, suppose that the settlement contract allows the pirate to use the product in perpetuity after they are caught, $T = \infty$. Using (8), a consumer's net payoff from stealing and settling once caught would be $B_i^2 = V_i - \theta_i X$. Notice that $\theta_i X$ is the discounted "price" that the consumer pays when they steal the good and settle the case once caught. If the consumer's type i was observable, then the firm could achieve perfect price discrimination by setting $X_i = V_i/\theta_i$ and $T_i = \infty$.[43] Indeed, with observable types, the firm would be indifferent between ex ante pricing with $P_i = V_i$ and ex post pricing with $X_i = V_i/\theta_i$. When consumer types are not observable, then the firm will use a combination of ex ante and ex post pricing.

Consider the incremental benefit that a type-i consumer gets from stealing the product versus buying it. Using the definitions of B_i^1 and B_i^2 in equations (3) and (8) we have

$$B_i^2 - B_i^1 = P - \theta_i \min\left\{D + \min\{V_i, P\}, X + e^{-rT}\min\{V_i, P\}\right\}. \tag{10}$$

Since $V_H > V_L$ and $\theta_H > \theta_L$ the right-hand side is decreasing in the consumer's type and so we have

$$B_L^2 - B_L^1 > B_H^2 - B_H^1.$$

In other words, if a consumer is given a choice between stealing the product and buying it, the L-types have a stronger incentive to steal than the H-types. So, it is *not* incentive compatible for the L-types to buy the product and the H-types to steal it.

The following Lemma summarizes the consumer's decision to purchase the product or steal it if settlement is not feasible.

Lemma 1. *Suppose that settlement is not feasible. When $D > v_i/\pi_i$, the type-i consumer buys the product if $P \leq V_i$ and does not consume it otherwise. When $D \leq v_i/\pi_i$, the type-i consumer buys the product if $P \leq \pi_i D/r$ and steals it otherwise.*[44]

This result is intuitive. Recall that v_i/π_i is the type-i consumer's *average accumulated surplus* from stealing the product. According to Lemma 1, if the sanctions for stealing are above this threshold, $D > v_i/\pi_i$, then stealing the good is a dominated strategy. The type-i consumer will therefore choose between buying it for price P and not consuming it. In contrast, if the sanctions for stealing are below this threshold, $D \leq v_i/\pi_i$, then not consuming the good is a dominated strategy. In this case the consumer will choose between buying the good and stealing it.

[43]Using (9), the cash payment $X_i = v_i/\pi_i + V_i$, the pirate's average accumulated surplus from stealing plus the pirate's value of a perpetual license.

[44]Note that the threshold $\pi_i D/r = \frac{D}{v_i/\pi_i} V_i \leq V_i$. If $P \in (\pi_i D/r, V_i]$ then the consumer, after stealing the product and paying D, subsequently pays P and continues to use it.

The next two subsections characterize the firm's optimal pricing and settlement policy, $\{P, X, T\}$. We will divide the analysis into two parts. First, we will analyze the case where property rights are strong, $D > \overline{D} = v_L/\pi_L$. Recall that $v_L/\pi_L \geq v_H/\pi_H$ by Assumption 1. Thus, when property rights are strong, the firm can deter all stealing by committing to a sufficiently high settlement offer or by refusing to settle altogether. Second, we will analyze the case where property rights are weak, $D \leq \overline{D}$. In this case, the possibility of stealing will constrain the firm's pricing and settlement policy.

3.1 Strong Property Rights

Suppose that the firm operates in a legal environment where property rights are strong, $D > \overline{D}$.

We begin by presenting a simple no-settlement benchmark. Lemma 2 establishes that with strong property rights and no possibility of out-of-court settlement, the firm implements the standard monopoly outcome.

Lemma 2. *(No-Settlement Benchmark 1.) Suppose that property rights are strong, $D > \overline{D}$, and firms cannot settle lawsuits. (1) If $\frac{\lambda_L}{\lambda_H} < \frac{V_H}{V_L} - 1$ then $P = V_H$, the H-types buy and L-types do not consume the product. Firm profits are $\lambda_H V_H$. (2) If $\frac{\lambda_L}{\lambda_H} \geq \frac{V_H}{V_L} - 1$ then $P = V_L$ and both types buy the product. Firm profits are $(\lambda_H + \lambda_L)V_L$.*

The results from the no-settlement benchmark (Lemma 2) are illustrated in Figure 1. When $\frac{\lambda_L}{\lambda_H}$ is low, so there are not many L-types in the consumer population, the firm sets $P = V_H$, sells to the H-types and excludes the L-types from the market. There is of course a deadweight loss associated with this outcome. When there are many L-types in the consumer population, so $\frac{\lambda_L}{\lambda_H}$ is high, then the firm sets $P = V_L$ and sells to everyone. This is socially efficient. Finally, note that the outcome described in Lemma 2 would be obtained in a static environment, too.

Although the firm has the power to implement the standard monopoly outcome by taking a "tough" no-settlement stance and setting $X \geq D$ and $T = 0$, the firm may prefer to take a "soft" stance when it comes to pirates. Since the two consumer types face different rates of being caught stealing and have different continuation values from a license of duration T, out-of-court settlement can be a valuable tool for price discrimination. By committing to settle lawsuits on advantageous terms (possibly bundled with a license for future use), the firm can *accommodate piracy* and generate additional revenue.[45]

[45] If the firm could not commit to the terms of settlement, it would have an ex post incentive to take each pirate to court to collect the higher court award. In anticipation, consumers would be deterred from stealing and firm profits would be lower.

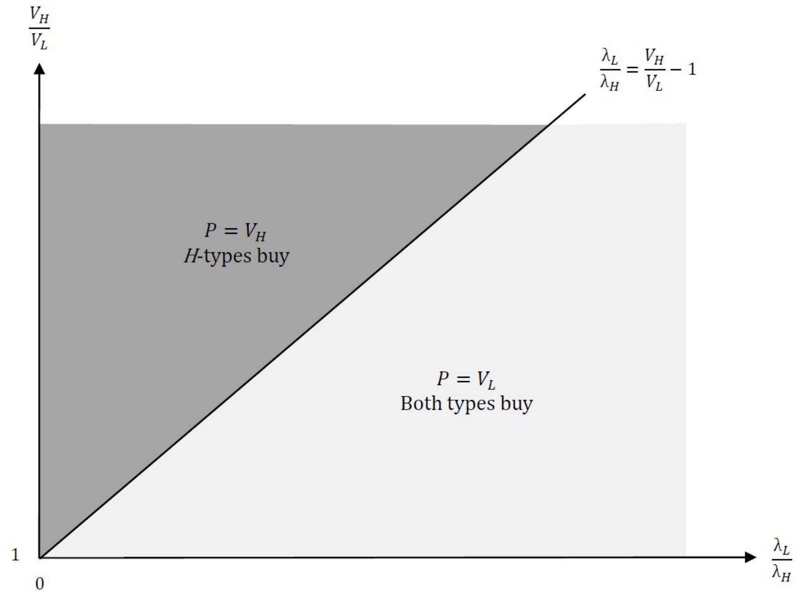

Figure 1: No-Settlement Benchmark (Strong Property Rights)

We now construct the most profitable policy under which the H-types buy and the L-types steal the product.[46] This policy must have the property that $P \in (V_L, V_H]$. First, if $P > V_H$, then the H-types would never purchase the product. If $P \leq V_L$ then the firm's profits are (weakly) lower than $(\lambda_L + \lambda_H)V_L$ (which the firm could achieve by choosing $P = V_L$ and refusing to settle lawsuits). We will construct the policy $\{P^*, X^*, T^*\}$ that solves the following program:

$$\text{Max}_{\{P,X,T\}} \quad \lambda_H P + \lambda_L \theta_L X \tag{11}$$

subject to

$$V_H - P \geq V_H - \theta_H(X + e^{-rT}P) \tag{12}$$

$$(1 - \theta_L e^{-rT})V_L - \theta_L X \geq 0 \tag{13}$$

$$P \in (V_L, V_H] \tag{14}$$

$$T \geq 0. \tag{15}$$

Condition (12) is the H-type's incentive-compatibility constraint and condition (13) is the L-type's individual-rationality constraint.

This program is not difficult to solve. Notice that the L-type's individual-rationality

[46]We show in the appendix that, when property rights are strong, the firm never implements a policy where both consumer types steal the product.

constraint in (13) must bind.[47] So we have

$$X = \left(\frac{1 - \theta_L e^{-rT}}{\theta_L} \right) V_L. \tag{16}$$

Using the definitions $\theta_L = \frac{\pi_L}{r+\pi_L}$ from (9) and $\overline{D} = \frac{v_L}{\pi_L} = \frac{rV_L}{\pi_L}$ we have

$$X = \overline{D} + (1 - e^{-rT})V_L. \tag{17}$$

In other words, the optimal cash settlement extracts the L-type's average accumulated surplus from stealing, $\overline{D}$, plus the L-type's discounted consumption value for the bundled license of duration T, $(1-e^{-rT})V_L$. Using (16), we rewrite the H-type's incentive-compatibility constraint (12) as

$$P \leq M(T)V_L, \tag{18}$$

where $M(T) \equiv \left(\frac{\theta_H}{1-\theta_H e^{-rT}} \right) \left(\frac{1-\theta_L e^{-rT}}{\theta_L} \right)$. It is not difficult to verify that $M(T)$ is decreasing in T and is strictly greater than 1.[48] We can rewrite the program as:

$$\text{Max}_{\{P,T\}} \quad \lambda_H P + \lambda_L \left(1 - \theta_L e^{-rT}\right) V_L \tag{19}$$

subject to

$$P \in (V_L, \min\{M(T)V_L, V_H\}] \tag{20}$$

$$T \geq 0. \tag{21}$$

The next proposition characterizes the firm's optimal policy.

Proposition 1. *(Settlement with Licensing 1.) Suppose that property rights are strong, $D > \overline{D}$. Define $\widetilde{T}$ by $M(\widetilde{T}) \equiv \frac{V_H}{V_L}$. Under the firm's optimal policy, H-types buy and L-types steal and settle if caught.*

1. *Suppose $\frac{V_H}{V_L} \leq \frac{\theta_H}{\theta_L}$. Then $P^* = V_H$, $X^* = \overline{D} + V_L$, and $T^* = \infty$.*
2. *Suppose $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$. If $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ then $P^* = V_H$, $X^* = \overline{D} + (1 - e^{-rT^*})V_L$, and $T^* = \widetilde{T} \in [0, \infty)$; otherwise, $P^* = \theta_H(\overline{D} + V_L)$, $X^* = \overline{D} + V_L$, and $T^* = \infty$.*

These results are illustrated in Figure 2. Consider the region below the horizontal line where the ratio of the consumers' valuations is smaller than the ratio of their discount factors, $\frac{V_H}{V_L} \leq \frac{\theta_H}{\theta_L}$. Strikingly, the firm achieves perfect price discrimination in this region. The H-types buy the product for $P^* = V_H$ and the L-types steal the product and settle out of court

[47] If it did not bind then the firm could raise X.

[48] When $T = 0$, $M(T) = \frac{\pi_H}{\pi_L} > 1$, and when $T = \infty$, $M(T) = \frac{\theta_H}{\theta_L} > 1$.

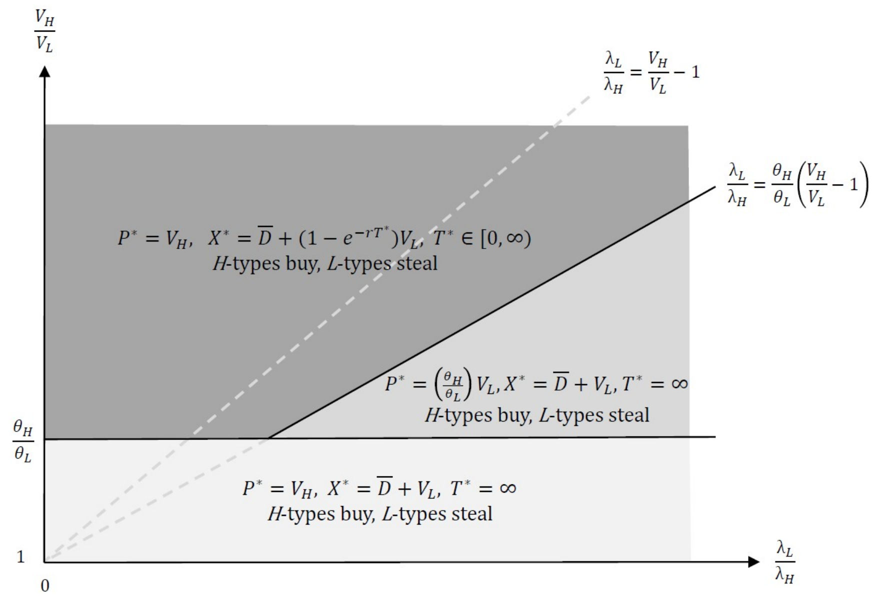

Figure 2: Settlement with Licensing (Strong Property Rights)

for a cash payment $X^* = \overline{D} + V_L$ bundled with a perpetual license, $T^* = \infty$. When viewed from an ex ante perspective, the discounted "price" paid by the L-types is $\theta_L X^* = V_L$. This mechanism is incentive compatible: since the H-types have a significantly higher discount factor (since they have a higher rate of apprehension), they will not mimic the L-types and steal the product.[49] Note that this outcome is socially efficient, since the L-types use the product in perpetuity.

Consider the regions above the horizontal line in Figure 2. If the ratio of the valuations is larger than the ratio of the discount factors, $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$, then perfect price discrimination is not possible. To prevent the H-types from mimicking the L-types, the firm must either reduce the price P^* or shorten the duration of the license T^*.[50] When there are relatively few L-types in the population, the firm will maintain the high market price, $P^* = V_H$, but limit the duration of the license, $T^* < \infty$. This policy extracts the full surplus from the H-types but distorts the consumption value of the L-types.[51] When there are many L-types in the population, the firm will charge a reduced market price, $P^* < V_H$, but include a perpetual license in the terms of settlement, $T^* = \infty$. This policy gives rents to the H-types but extracts the full surplus from the L-types.

It is interesting to compare our results to the standard screening model where a firm sells

[49] That is, condition (12) is not binding.

[50] This implies that condition (12) must be binding.

[51] The pirates privately observe their types. $T^* < \infty$ prevents the H-types from mimicking the L-types. If the firm could observe the pirates' types after catching them, then the firm would price discriminate and offer $X_L = \overline{D} + V_L$ and $T_L = \infty$ to the L-types and $X_H = \overline{D} + V_H$ and $T_H = \infty$ to the H-types. Distorting T^* would be unnecessary.

two versions of a product to two types of consumer with different preferences for product quality. In the standard screening model, to prevent the H-type consumers from mimicking the L-types, the optimal mechanism distorts the quality of the low version.[52] Our setting also involves two "versions": a legitimate version for the H-types (paid for ex ante) and a stolen version for the L-types (paid for ex post). However, our H-types are deterred from mimicking the L-types and stealing because the H-types are caught at a higher rate, not because the stolen version is of lower quality.[53] To see this clearly, suppose that the H-types mimic the L-types and steal. After settling out of court and finishing the license of duration T, the H-types pay $P^* \in (V_L, V_H]$ for a new perpetual license. Thus, the two versions — the legitimate version and the stolen version — provide the same "quality" (i.e. the duration of product use) to the H-types.[54]

Based on Lemma 2 and Proposition 1, we have the following welfare comparison.

Corollary 1. *Suppose that property rights are strong, $D > \overline{D}$. Compared to the no-settlement benchmark, settlement with licensing strictly increases social welfare when $\frac{\lambda_L}{\lambda_H} < \frac{V_H}{V_L} - 1$, strictly decreases social welfare when $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$ and $\frac{\lambda_L}{\lambda_H} \in \left(\frac{V_H}{V_L} - 1, \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)\right)$, and has no effect otherwise.*

Corollary 1 tells us that out-of-court settlement may either raise or lower social welfare compared to the no-settlement benchmark.[55]

To see why, consider the no-settlement benchmark. First, in the region above the diagonal line in Figure 1, the firm sets $P = V_H$ and excludes the L-types from the market (Lemma 2). By contrast, in Figure 2 when settlement is allowed, the firm accommodates the L-types with a soft settlement policy (see also Proposition 1). Thus, in this region, settlement strictly raises social welfare by reducing the deadweight loss. Second, in the region below the diagonal line in Figure 1, the firm sets $P = V_L$ and both consumer types buy the product. In Figure 2, when settlement is allowed, the L-types steal the product and get a license with duration $T^* > 0$. If the license is of limited duration, $T^* < \infty$, then settlement strictly lowers social welfare by creating a deadweight loss.[56]

[52] For example, see Chapter 2 in Salanie (1997).

[53] For the L-types, the "quality" of the stolen version depends on T, the duration specified in the settlement contract.

[54] In the standard screening model, the H-types get higher utilities from the low quality product than the L-types, so the H-types receive rents under the optimal mechanism. By contrast, in our model, the H-types are more likely to be caught and therefore may not get higher utilities from stealing than the L-types. So, the optimal policy does not need to distort the quality (T) and leave rents to the H-types at the same time.

[55] Our working paper also explores settings where private settlement contracts may not include licenses ($T = 0$). We show that settlement with licensing (weakly) raises social welfare as compared to cash-only settlement.

[56] If $T^* = \infty$ then the L-type uses the product at every active moment of their lives and social welfare remains the same.

3.2 Weak Property Rights

Now suppose that the firm operates in a legal environment where property rights are weaker, $D \leq \overline{D}$.

As in the previous subsection, we begin with a benchmark where the firm is unable to settle lawsuits out of court. If the firm sets $P > V_L$ in this environment, then the L-types will steal because their net valuation from stealing would be positive, $(1-\theta_L)V_L - \theta_L D \geq 0$. As shown in the next Lemma, when property rights are weak, stealing is not fully deterred in equilibrium: the L-types steal the product and the H-types may steal as well.

Lemma 3. *(No-Settlement Benchmark 2.) Suppose that property rights are weak, $D \leq \overline{D} = v_L/\pi_L$, and the firm cannot settle lawsuits. Firm profits increase in D. There exist thresholds $\underline{D} < \widehat{D} \leq \overline{D}$ where*

$$\underline{D} = v_L/\pi_H \quad \text{and} \quad \widehat{D} = v_H/\pi_H. \tag{22}$$

1. *$D \in \left[\widehat{D}, \overline{D}\right]$. If $\frac{\lambda_L}{\lambda_H} < \frac{1}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ then $P = V_H$; otherwise, $P = V_L$. The H-types buy and L-types steal.*
2. *$D \in \left[\underline{D}, \widehat{D}\right)$. If $\frac{\lambda_L}{\lambda_H} < \frac{1}{\theta_L}\left[\frac{\theta_H(D+V_H)}{V_L} - 1\right]$ then $P = V_H$ and both types steal; otherwise, $P = V_L$ and the H-types buy and L-types steal.*
3. *$D < \underline{D}$. If $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ then $P = V_H$; otherwise, $P = V_L$. Both types steal.*

Recall that $\widehat{D} = v_H/\pi_H$ is the type-H consumer's average accumulated surplus from stealing (if the consumer does not exit the market). Lemma 1 implies that, for the type-H consumer, stealing is a dominated strategy if $D > \widehat{D}$ while not consuming the good is a dominated strategy if otherwise. If the property right is even weaker such that $D < \underline{D} = v_L/\pi_H$, then the type-$H$ consumer steals the good even when the market price equals V_L.

Consider the first case of Lemma 3 where $D \in \left[\widehat{D}, \overline{D}\right]$. When there are relatively many H-types in the population, so $\frac{\lambda_L}{\lambda_H}$ is small, the firm sets $P = V_H$. In this case, the H-types buy the product for price P and L-types steal and pay D to the firm when caught. There is a deadweight loss associated with this outcome, as the L-types are excluded from the market after paying damages D. When $\frac{\lambda_L}{\lambda_H}$ is large, the firm sets $P = V_L$. The H-types buy the product and the L-types steal it and, when caught, pay $D + V_L$. Note that in this case, the L-types purchase the product at price $P = V_L$ after being caught stealing.

Now consider the second case of of Lemma 3 where $D \in \left[\underline{D}, \widehat{D}\right)$. If the firm sets $P = V_H$ then both consumer types would steal the product. When there are sufficiently many H-types in the consumer population, then setting $P = V_H$ is in fact the firm's optimal strategy. Although both consumer types obtain rents from stealing, the firm will fully extract the H-type consumer's future surplus after the H-type is caught (since the H-type purchases the good after being apprehended). When there are many L-types in the population, the firm's optimal strategy is to charge $P = V_L$. In this case, the H-types buy the good and the L-types steal until caught and pay $D + V_L$.

Finally, if property rights are very weak, $D < \underline{D}$, then the firm charges either $P = V_L$ or $P = V_H$ and both consumer types steal the product. The firm receives damage payments D when a consumer is stealing, and in addition obtains revenues from the subsequent sales. If $P = V_L$ then both consumer types purchase the product after they are apprehended for stealing, and if $P = V_H$ then only the H-type purchases.

We now turn our attention to out-of-court settlement. As in the previous subsection with strong property rights, we will show that settlement (possibly bundled with a license) enhances the firm's ability to price discriminate. We consider three regions of the damage award D.

To begin, suppose that $D \in \left[\widehat{D}, \overline{D}\right]$. We show in the appendix that the firm will implement a pricing and settlement policy where the H-types buy the product and the L-types steal it. Suppose hypothetically that $P = V_H, X = D + V_L$ and $T = \infty$. Note that $X = D + V_L$ is the very most that the firm could hope to extract from the L-type, since the L-type is willing to pay D to avoid the trial plus an additional amount V_L for a perpetual license with $T = \infty$. This policy also succeeds in extracting the full surplus from the H-types when $V_H \leq \theta_H(D + V_L)$, since the H-types would not have incentive to mimic the L-types and steal the product in this case. So, when $V_H \leq \theta_H(D + V_L)$, this policy extracts the highest possible rent from both consumer types.

If $V_H > \theta_H(D + V_L)$, then the firm cannot extract the highest possible rent from both consumer types. To prevent the H-types from mimicking the L-types, the firm must either reduce the price P or shorten the duration of the license T. Reducing the license duration allows the firm to extract the full surplus from the H-types but decreases the L-types' total consumption value. If there are relatively few L-types in the population, the former effect dominates and the firm would maintain the high market price but offer a license of limited duration. If there are many L-types in the population, the latter effect dominates and therefore the firm would offer a license of infinite duration and reduce the market price.

The next proposition characterizes the firm's optimal policy.

Proposition 2. *(Settlement with Licensing 2.) Suppose* $D \in \left[\widehat{D}, \overline{D}\right]$. *Define* $\widehat{T}$ *by* $V_H \equiv \frac{\theta_H}{1-\theta_H e^{-r\widehat{T}}}[D + (1 - e^{-r\widehat{T}})V_L]$. *H-types buy and L-types steal and settle if caught.*

1. *Suppose* $V_H \leq \theta_H(D + V_L)$. *Then* $P^* = V_H$, $X^* = D + V_L$ *and* $T^* = \infty$.

2. *Suppose* $V_H > \theta_H(D+V_L)$. *If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* = V_H$, $X^* = D + (1 - e^{-rT^*})V_L$, *and* $T^* = \widehat{T} \in [0, \infty)$; *otherwise,* $P^* = \theta_H(D + V_L)$, $X^* = D + V_L$ *and* $T^* = \infty$.

Now suppose that property rights are even weaker, so $D \in [\underline{D}, \widehat{D})$. Since $D < \widehat{D}$, to prevent the H-types from stealing, the firm would have to set a sufficiently low market price. As shown in the following proposition, if there are many H-types in the population, the firm sets $P^* = V_H$ and both types steal. Following apprehension, the H-types purchase the product and the L-types stop using it. If there are relatively few H-types, the firm sets $P^* < V_H$ and the H-types buy the product and the L-types steal it and settle out of court and receive perpetual licenses. Settlement with licensing helps the firm to monetize its property rights and facilitates price discrimination.

Proposition 3. *(Settlement with Licensing 3.) Suppose* $D \in [\underline{D}, \widehat{D})$.

1. *If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* = V_H$, $X^* = D$, *and* $T^* = 0$. *Both types steal and settle if caught.*

2. *If* $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* = \theta_H(D + V_L) \geq V_L$, $X^* = D + V_L$ *and* $T^* = \infty$. *H-types buy and L-types steal and settle if caught.*

Finally, suppose that property rights are very weak, so $D < \underline{D}$. To prevent the H-types from stealing, the firm would have to set an extremely low market price. It is more profitable for the firm to accommodate stealing by both consumer types and to monetize the property rights through settlement contracts.

Proposition 4. *(Settlement with Licensing 4.) Suppose* $D < \underline{D}$. *Both types steal and settle if caught.*

1. *If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* = V_H$, $X^* = D$, *and* $T^* = 0$.

2. *If* $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* \geq V_L$, $X^* = D + V_L$, *and* $T^* = \infty$.[57]

This result is intuitive. When property rights are very weak and there are many H-types in the population, the firm sets $P^* = V_H$, $X^* = D$, and $T^* = 0$. Both types steal until they are apprehended. Once caught, the H-types purchase the perpetual license at price $P^* = V_H$ and the L-type do not consume further. When there are many L-types, the firm sets $X^* = D + V_L$ and $T^* = \infty$. When caught, both types receive perpetual licenses as part of the settlement deal.

Comparing Propositions 2, 3, and 4 to Lemma 3, one can see that, when property rights are weak, settlement with licensing weakly increases social welfare compared to the no-settlement benchmark.[58]

[57] If $P = V_L$, there exists a continuum of equivalent policies with $X = D + (1 - e^{-rT})V_L$ *for any* $T \geq 0$.

[58] Our working paper version also shows that, when property rights are weak, cash-only settlement (i.e. $T = 0$) leads to the same social welfare as in the no-settlement benchmark.

Corollary 2. *Suppose that property rights are weak ($D \leq \overline{D}$). Compared to the no-settlement benchmark, settlement with licensing weakly increases social welfare.*

Given $D \leq \overline{D}$, we can verify that $\widehat{T}$ (as defined in Proposition 2) increases in D and is weakly lower than $\widetilde{T}$ (as defined in Proposition 1).[59] Then comparing Proposition 1, 2, 3, and 4, one can see that, the duration of licenses bundled with settlement is (weakly) longer when property rights become stronger (i.e. D is larger). Intuitively, when the court-ordered damage award, D, becomes larger, the H-types have (weakly) less incentive to mimic the L-types, so that the firm could raise the duration T of the license bundled with settlement as well as the settlement amount X to extract more rents from the L-types.

Corollary 3. *The duration of licenses in the optimal settlement policy, T^*, weakly increases in D.*

This section showed that settling lawsuits with pirates, and bundling cash payments with future purchase agreements, can be a profitable business strategy. Out-of-court settlement provides a second way for firms to monetize their property rights and facilitates price discrimination. Compared to the no-settlement benchmark, settlement has an ambiguous impact on social welfare if property rights are strong (court-awarded damages D are sufficiently high) and raises social welfare if property rights are weak.

4 Discussion

4.1 Limited Commitment

The baseline model assumes that the firm can commit to its settlement policy and there is no private renegotiation between the firm and stealing consumers over the settlement contract. Similar insights may be obtained when firm lacks full commitment power and/or the settlement contract cannot be observed by the public.[60] In this subsection, we assume that the firm commits to the market price P but cannot commit to the terms of settlement (X, T).[61] After a consumer is caught stealing, the firm makes a private take-it-or-leave-it settlement offer (X, T) to the consumer.[62] We will demonstrate that out-of-court settlement facilitates price discrimination, assuming that property rights are not too strong.

[59] Note that $\frac{\theta_H}{1-\theta_H e^{-rT}}[D + (1 - e^{-rT})V_L]$ is less than $M(T)V_L$ given $D \leq \overline{D}$. Since $M(T)V_L$ decreases in T, by the definitions, we have $\widetilde{T} \geq \widehat{T}$.

[60] If settlement negotiations were public, firms might obtain commitment through reputation.

[61] Since the environment is stationary and P is observable, the firm can commit to the price. Similar to the baseline model, the perpetual license can be replaced by short-term licenses and there exist other equivalent settlement contracts that require the pirates to buy licenses at discounted prices.

[62] We maintain Assumption 1 $\left(\frac{v_H}{\pi_H} \leq \frac{v_L}{\pi_L}.\right)$.

Suppose that $P^* \in (V_L, V_H]$ and consider the firm's sequentially rational settlement strategy after catching a pirate. After catching a stealing consumer, the firm updates its prior belief about the consumer's type. If the firm places a high posterior weight on the consumer being an H-type, the firm would offer to settle for $(X, T) = (D, 0)$. Both consumer types would accept this settlement offer, of course, and the H-type would subsequently purchase a license at the market price P^*. In contrast, if the firm places a high posterior weight on the consumer being an L-type, then the firm would bundle a license agreement with the settlement, $(X, T) = (D + V_L, \infty)$. Including the perpetual license allows the firm to extract the full consumption value from the L-types. Notice that the L-type's continuation valuation is $-D$ after being caught stealing, since the L-type receives zero consumer surplus from continued use.

The next proposition summarizes the firm's optimal pricing and settlement policy when the firm has limited commitment power. The formal proof is presented in the appendix.

Proposition 5. *(Limited Commitment.) Suppose that the firm can make private settlement offers after catching a consumer for stealing. The firm's optimal strategy has the following properties.*

1. $D > \overline{D}$. *Neither consumer type steals. The equilibrium is the same as in Lemma 2.*
2. $D \in [\widehat{D}, \overline{D}]$. *Then* $P^* = \min\{V_H,\ \theta_H(D + V_L)\}$, $X^* = D + V_L$, *and* $T^* = \infty$. *The H-types buy and L-types steal and settle if caught.*
3. $D \in [\underline{D}, \widehat{D})$. *The optimal price and settlement offers are the same as in Proposition 3.*
4. $D < \underline{D}$. *The optimal price and settlement offers are the same as in Proposition 4.*

Suppose that property rights are strong, so $D > \overline{D}$. The firm's optimal pricing strategy is the same as in the no-settlement benchmark in Lemma 2 and there is no stealing in equilibrium. In equilibrium, the firm chooses either $P^* = V_H$ or $P^* = V_L$, depending on the proportion of H-types in the consumer population, and subsequently offers to settle lawsuits for $(X^*, T^*) = (D, 0)$ if a consumer is caught stealing. This settlement offer, which is made off the equilibrium path, is supported by the firm's posterior belief that only H-types would steal.[63] Note that the L-type would never steal because their net valuation from stealing is negative, $(1 - \theta_L)V_L - \theta_L D < 0$.

Suppose property rights are in an intermediate range, $D \in [\widehat{D}, \overline{D}]$. In this case, the H-types buy the product for $P^* > V_L$ and L-types steal it in equilibrium. Believing (correctly)

[63] There are other posterior beliefs and off-the-equilibrium path settlement offers that support this equilibrium.

that any consumer who steals is an L-type, the firm offers to settle for $(X^*, T^*) = (D + V_L, \infty)$ and extracts all surplus from the L-type consumers. Thus, settlement with licensing facilitates price discrimination. Recall that in our baseline model where the firm could commit to its settlement policy, the firm may include a license of finite duration in the settlement contract (Proposition 2). Since commitment is not possible here, the firm extends the duration of the license to extract more surplus from the L-type consumers ex post.

When property rights are even weaker, $D \in \left[\underline{D}, \widehat{D}\right)$ or $D < \underline{D}$, the renegotiation-proof settlement polices are the same as those in Propositions 3 and 4 in the baseline model. It is also easy to see that the welfare impacts of the settlement policies are the same as those identified in Corollary 2.

4.2 Network Effects

The baseline model shows that firms may adopt policies that are soft on pirates and use settlement bundled with licensing as an instrument for price discrimination. In other words, settlement provides an alternative channel for firms to monetize their property rights. Price discrimination is not the only reason why firms might accommodate pirates. Accommodating pirates can help a firm to expand the market, create direct network effects among consumers, and encourage the development of complementary products and services. This may be particularly true for the software industry where obtaining (and maintaining) a critical mass of consumers is widely recognized as an important driver of success.

In the online appendix, we extend our baseline model to include direct network effects where each consumer's gross valuation is proportional to the total number of users of the product.[64] As in the baseline model, we assume that the firm commits to a policy $\{P, X, T\}$ and then characterize the stationary equilibrium where we let $N(T)$ represent the total number of users at each moment of time. Intuitively, a longer license duration T increases the number of consumers using the product, $N(T)$, and increases the consumers' willingness to pay. A type-i consumer's discounted gross value from using the product is $N(T)V_i$, for $i = H, L$.

This extension delivers two clear observations. First, the introduction of network effects does not change our insight that settlement with licensing facilitates price discrimination. Recall that, in the baseline model without network effects, it is optimal for the firm to choose a policy with $T = \infty$ when $\frac{V_H}{V_L}$ is sufficiently small or $\frac{\lambda_L}{\lambda_H}$ is sufficiently large. With network effects, each consumers' gross value increases in T. Therefore, given the same parameter values, it is still optimal for the firm to choose the policy with $T = \infty$, under which the H-types buy and the L-types steal the product. Therefore the optimal settlement policy has

[64]For simplicity, we consider the case where property rights are strong.

the dual benefit of implementing price discrimination and generating network effects.

Second, network effects motivate the firm to choose a license duration T that is (weakly) longer than that in the baseline model. As shown in the baseline model, the firm sometimes may choose a limited license duration $T < \infty$ to facilitate price discrimination. However, given network effects, a longer duration would increase consumers' gross value and therefore raise the firm's profit. Given this trade-off between network effects (creating value) and price discrimination (capturing value), the firm has incentives to prolong the duration of the license bundled with the settlement contract.[65]

4.3 Entry Deterrence

Our baseline model assumed that the firm enjoyed monopoly power and was insulated from outside competition. We showed that the firm might structure its settlement contracts with pirates to include licenses for future use. When facing potential entrants, the firm may have an incentive to extend the license duration in the settlement contracts to lock in consumers and create barriers to entry.[66] Thus, settling lawsuits with pirates can have important anticompetitive effects.

To illustrate the potential anticompetitive effects of settlement contracts, the online appendix presents a simple extension with one strategic entrant and consumers who have limited awareness of their outside options.[67] The potential entrant, E, arrives according to an exogenous Poisson process with arrival rate β. Upon arrival, the entrant decides whether to enter the market or not. The lump-sum entry cost is $K > 0$. The entrant's product generates an instantaneous value $v_i + u$ for a type-i consumer, where $u > 0$ and we let $U = u/r$.

Suppose that consumers who are using the product legally are completely unaware of the entrant's product, both before and after entry. These unaware consumers include both users who purchased a (perpetual) license for price P and also users who previously settled piracy charges and are currently using the product under a license of duration T. These unaware consumers are effectively "locked-in" and will not switch to the entrant.[68]

[65]Specifically, we show in the online appendix that the firm chooses a license duration strictly longer than that in the baseline model when property rights are strong, consumers' exit rate α is sufficiently small, and $\frac{\lambda_L}{\lambda_H}$ is in an intermediate range.

[66]A large literature studies the anticompetitive effects of various contracts (for a few examples, see Aghion and Bolton, 1987; Spier and Whinston, 1995; Rasmusen et al., 1993; Simpson and Wickelgren, 2007; Ide et al., 2016). This literature, however, has not considered the possibility for (bundled) settlement contracts to have deterrence effects as shown in this paper.

[67]For simplicity we focus on the case of strong property rights. In this extension, both the incumbent's and the entrant's strategies are stationary, which simplifies analysis. If consumers are fully aware of their outside options, the anticompetitive effects still exist but the firms' strategies become non-stationary.

[68]Consumers' limited awareness is common in practice. For example, consumers who are using the product legally would pay less attention to advertisements from entrants; they may also face high switching

The entrant can attract the other "free" consumers: (1) consumers who are new to the market; (2) users who are stealing the product but have not yet been caught; and (3) consumers who used the incumbent's product in the past but are no longer using it. The latter category includes users who settled with the incumbent and stopped usage after the license of duration T expired. These consumers are aware of the entrant's product, and will purchase the product from whichever seller delivers the highest consumer surplus.

As shown in the baseline model without a threat of entry, the incumbent may offer a license of limited duration $T < \infty$. However, when facing an entry threat, the incumbent has incentives to "lock in" more consumers by extending the duration of the license.[69] In the online appendix, we characterize the conditions under which the incumbent chooses a longer duration to deter the entrant.

A natural question is whether imposing a cap on the duration of licenses bundled with settlement contracts (or banning bundled settlements altogether) can raise social welfare.[70] Although such regulations would lower the barriers to entry, they may also reduce the consumption duration of consumers before the entrant arrives. So the overall welfare impact is ambiguous. However, in the scenario where the incumbent chooses a license duration which is limited but strictly longer than that in the baseline model, a slightly shorter license duration raises welfare, as it accommodates entry but does not reduce the pre-entry consumption duration by too much.

4.4 Recidivism

The baseline model assumes that the two types of consumers have different probabilities of being caught the first time, but both will be caught for sure (and immediately) the second time. Thus, there is no recidivism in our baseline model. Our insight about settlement as a tool for price discrimination is robust to recidivism. Suppose that consumers, after being caught stealing and settling the lawsuit, can steal the product again and enjoy the same rate of detection as they did the first time.[71] The firm commits to a policy $\{P, X, T\}$, where the settlement offer (X, T) is available for consumers who are caught each time.[72] As in the

costs. Alternatively, consumers may have positive but negligible search costs before learning which firms are available on the market and observing their prices.

[69] Our model assumes a stationary market. If the mass of consumers is growing over time, it would be harder for the incumbent to deter the entrant because more "free" consumers enter the market over time. In this case, to deter entry, the incumbent might include a license of even longer duration.

[70] The working paper version of our paper also considers the case where parties are prohibited from bundling licenses with their settlement contracts. This restriction corresponds to $T = 0$.

[71] This is similar to a "catch and release" technique in recreational fishing, where fish are caught, unhooked, and thrown back into the water. Note that this technology creates more informative signals and can more easily separate the two consumer types.

[72] As before, the firm observes whether a consumer has been caught stealing or not, but does not observe the duration of the stealing or the consumer's history of settlements.

baseline model, we assume that given two revenue-equivalent policies, one where consumers steal and another where consumers purchase or license the product, the firm chooses the policy where consumers license the product.[73]

If a type-i consumer steals the product, then after settling with the firm and finishing the license of duration T, the consumer should have incentives to steal again. So, recidivism gives the firm more precise information about consumer types and enhances its ability to price discriminate, without reducing stealing consumers' consumption duration. Consistent with this intuition, we show in the online appendix that, when property rights are strong, there exists a continuum of policies that can implement perfect price discrimination, under which the H-types buy at the price $P = V_H$ and the L-types steal the product repeatedly. Among these incentive-compatible policies, the firm would choose the one with the longest license duration (which may be finite).

4.5 Concave Detection Rate

In the baseline model, we assume that the detection rate is a weakly convex function of consumer type, so the type-L consumer's *average accumulated surplus* from stealing until caught is larger than the type-H consumer's, $\frac{v_H}{\pi_H} \leq \frac{v_L}{\pi_L}$ (Assumption 1). The insights regarding the use of settlement with licensing for price discrimination are robust even if $\frac{v_H}{\pi_H} > \frac{v_L}{\pi_L}$. The online appendix presents the optimal settlement policy when $\frac{v_H}{\pi_H} > \frac{v_L}{\pi_L}$ and property rights are strong. This extension leads to the following observations.

First, since the H-types have a much higher consumption value than the L-types, different from the baseline model, the firm may have incentives to refuse settlement and sell the product only to the H-types. This is true when there are relatively few L-types in the consumer population ($\frac{\lambda_L}{\lambda_H}$ is small).

Second, when $\frac{\lambda_L}{\lambda_H}$ is in an intermediate range, the firm finds it optimal to settle lawsuits without licensing ($T = 0$) and charge the high market price $P = V_H$. Since the detection rate for the H-types is relatively small ($\frac{v_H}{\pi_H} > \frac{v_L}{\pi_L}$), the H-types have incentives to mimic the L-types and steal the product. That is, both types steal the product. However, since the two types of consumers face different detection rates, in expectation they pay different prices and, after being caught, the H-types buy the product and the L-types stop using it.

Finally, when there are many L-types in the consumer population ($\frac{\lambda_L}{\lambda_H}$ is large), it is optimal for the firm to charge a lower market price $P < V_H$ but include a perpetual license ($T = \infty$) in the settlement terms. Then, the H-types buy the product while the L-types steal it and settle out of court if caught. This scheme facilitates price discrimination.

[73] This would be the equilibrium outcome with positive costs of litigation and settlement.

5 Conclusion

Piracy poses a major challenge for firms whose products are protected by intellectual property rights. Many consumers choose to use pirated software, photographs, and other products rather than paying for the rights. In response, many firms have been taking legal actions to enforce their property rights, bringing lawsuits against known pirates in an attempt to monetize their intellectual property. Interestingly, the out-of-court settlements between the right-holders and the pirates may be bundled with agreements by the pirates to purchase the product in the future. In addition to specifying cash payments, the settlement contracts may specify the monetary value of future purchases, the duration of the future license agreement, or both.[74] Our paper developed a theoretical framework to shed light on these settlement contracts and to evaluate their welfare implications.

From a positive or descriptive perspective, our model can help to explain real-world business practices of companies like Microsoft, Monsanto, Getty Images, and trade associations such as the Business Software Alliance (BSA). We show that private enforcement actions may allow firms to monetize their intellectual property and generate an additional stream of revenue. Firms may adopt out-of-court settlement policies that are "soft" on pirates, offering to settle for less than the court-ordered damage award and bundling settlement with licenses for future use. We show that these settlement policies are profitable because they enhance the firm's ability to price discriminate when consumers are heterogeneous.

Our model also has implications for the duration and scope of settlement agreements. First, in jurisdictions where property rights are stronger in the sense that court-ordered damage awards are higher, settlement contracts are more likely to include licenses for future use. Second, cash settlements will be larger, and the associated license duration longer, when there is either a greater proportion of low-value consumers in the market or when the willingness to pay of these low-value consumers is relatively larger. License duration will also be longer when there are direct network externalities among consumers or when there is a threat of market disruption by new entrants.[75]

From a normative perspective, we show that, as compared to the no-settlement benchmark, bundling settlement agreements for past infringement with licenses or purchase agreements for future use could either create social value by expanding the market to include an otherwise excluded group, or could destroy social value by limiting market access by an oth-

[74] As documented in the introduction, Monsanto's settlement agreement with one hundred U.S. farmers specified a six-year license with future payments of $1.1 million. In the software industry, more and more firms tend to "rent" their products to consumers, so settlement could be combined with rental contracts.

[75] Chinese consumers' willingness to pay for non-pirated software has increased in recent years. See https://technode.com/2017/09/26/why-chinese-are-starting-to-pay-for-software/. Laws on piracy vary significantly across countries. See https://blog.redpoints.com/en/why-is-piracy-such-a-problem-in-europe. These time and cross-country variations may facilitate the empirical study of settlement contracts.

erwise active group. Moreover, bundling long-term licenses with settlements may enhance the direct network effects among consumers, but may also put more efficient entrants at a strategic disadvantage and could deter or delay their entry into the market.[76]

Our model abstracted away from strategic interactions among downstream users. It would be meaningful for future research to examine settlement policies when users of the product, including both purchasing consumers and pirates, are downstream competitors. Stealing products could give pirates a competitive advantage over legitimate users, who in turn would be less willing to pay for the products. In this environment, the firm may commit to a tough settlement policy to mitigate downstream cannibalization and support higher upstream markups. Future research could also examine how settlement policies affect ex ante innovation incentives.[77]

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[76]The potential impact of IPR enforcement and settlement has received attention from policymakers, including antitrust authorities in China. There has been related policy debate in the United States over the "customer suit exception" doctrine, which restricts patent holders from filing lawsuits against downstream users who have bought pirated products from illegal manufacturers (see Love and Yoon, 2013).

[77]In 2017 the US Department of Justice published the revised Antitrust Guidelines for the Licensing of Intellectual Property, including the concern about package licensing (bundling multiple licenses in one single contract). See https://www.justice.gov/atr/IPguidelines/download. Although not mentioned in the Guidelines, out-of-court settlement with licensing could be regarded as one special form of package licensing.

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Appendix A

This appendix contains the proofs of the main results. The technical details of the extensions in Subsections 4.2-4.5 may be found in Online Appendix B. Note that the following thresholds defined in Section 3 can be rewritten as $\overline{D} = v_L/\pi_L = \frac{1-\theta_L}{\theta_L}V_L$, $\widehat{D} = v_H/\pi_H = \frac{1-\theta_H}{\theta_H}V_H$, and $\underline{D} = v_L/\pi_H = \frac{1-\theta_H}{\theta_H}V_L$.

Proof of Equation (8). Let Z denote the consumer's continuation payoff at the moment that they are caught stealing as given in equation (7):

$$Z = V_i - \min\left\{D + \min\{V_i, P\}, X + e^{-rT}\min\{V_i, P\}\right\}.$$

Suppose that a type-i consumer who is active at time $t = 0$ steals the product and will be caught at time $\tau > 0$. In this scenario, the consumer's net benefit from stealing (including the possibility of exiting before or after time τ) is:[78]

$$\int_0^\tau v_i e^{-rt}dt + Ze^{-r\tau}.$$

The first term is the value that the consumer enjoys while stealing the product from time 0 through time τ. The second term is the present value of the consumer's surplus after being caught at time τ. This may be rewritten as

$$V_i - e^{-r\tau}\min\left\{D + \min\{V_i, P\}, X + e^{-rT}\min\{V_i, P\}\right\}.$$

Recall that the moment a type-i consumer is caught stealing, τ, is exponentially distributed with density $f_i(\tau) = \pi_i e^{\pi_i\tau}$. So, the net valuation for a type-i consumer who steals the product is

$$\int_0^\infty \left[V_i - e^{-r\tau}\min\left\{D + \min\{V_i, P\}, X + e^{-rT}\min\{V_i, P\}\right\}\right]\pi_i e^{-\pi_i\tau}d\tau.$$

This expression simplifies and is equivalent to equation (8).

Proof of Lemma 1. First, suppose $D > v_i/\pi_i$. Using the definition of θ_i in (9) and $v_i = rV_i$, this is equivalent to $D > \frac{1-\theta_i}{\theta_i}V_i$. Using (8), if $P \leq V_i$, consumer prefers to buy the product than steal it if $V_i - P \geq V_i - \theta_i(D + P)$ or equivalently $D \geq \frac{1-\theta_i}{\theta_i}P$. This is true since $D > \frac{1-\theta_i}{\theta_i}V_i$ and $V_i \geq P$. If $P > V_i$, the consumer would not purchase after being caught stealing. The consumer prefers to not consume the product than steal when

[78]The benefit is the sum of $\int_0^\tau\left(\int_0^\eta v_i e^{-\rho t}dt\right)\alpha e^{-\alpha\eta}d\eta$ (the value if exiting before time τ) and $\int_\tau^\infty\left(\int_0^\tau v_i e^{-\rho t}dt + Ze^{-\rho\tau}\right)\alpha e^{-\alpha\eta}d\eta$ (the value if exiting after time τ).

$(1-\theta_i)V_i - \theta_i D < 0$ which is true because $D > \frac{1-\theta_i}{\theta_i}V_i = v_i/\pi_i$.

Now suppose $D \leq v_i/\pi_i = \frac{1-\theta_i}{\theta_i}V_i$. Rearranging gives $(1-\theta_i)V_i - \theta_i D \geq 0$. This implies that the consumer's payoff from stealing the product in (8) is non-negative, $V_i - \theta_i(D + min\{V_i, P\}) \geq (1-\theta_i)V_i - \theta_i D \geq 0$. Therefore the consumer chooses between stealing the product and buying it. If $P > V_i$ the consumer gets negative surplus from buying and therefore steals. If $P \leq V_i$ the consumer steals (and subsequently buys after being caught) if $V_i - \theta_i(D+P) < V_i - P$ or $P > \frac{\theta_i}{1-\theta_i}D$. Using (9) this is equivalent to $P > \pi_i D/r$.

Proofs of Lemmas 2 and 3. We will consider four cases, including both the case with strong property rights ($D > \overline{D} = v_L/\pi_L = \frac{1-\theta_L}{\theta_L}V_L$) and three cases with weak property rights ($D \leq \frac{1-\theta_L}{\theta_L}V_L$). Note that $\widehat{D} = v_H/\pi_H = \frac{1-\theta_H}{\theta_H}V_H$, and $\underline{D} = v_L/\pi_H = \frac{1-\theta_H}{\theta_H}V_L$. Assumption 1 implies $\underline{D} < \widehat{D} \leq \overline{D}$, or equivalently, $\frac{1-\theta_H}{\theta_H}V_L < \frac{1-\theta_H}{\theta_H}V_H < \frac{1-\theta_L}{\theta_L}V_L$.

(1) Suppose $D > \overline{D} = v_L/\pi_L$. Assumption 1 implies $D > v_H/\pi_H$. Using Lemma 1, neither consumer type will steal. Consider three regions for the price P.

1. $P \leq V_L$. Both types buy the product and firm profits are $(\lambda_H + \lambda_L)V_L$.
2. $P \in (V_L, V_H]$. The H-types buy and L-types do not consume. Firm profits are $\lambda_H P$. The price that maximizes profits in this range is $P = V_H$, and firm profits are $\lambda_H V_H$.
3. $P > V_H$. Neither type buys and firm profits are zero.

The firm would never set $P > V_H$ (region 3) because profits would be zero. The profits in regions 1 and 2 are equal when $(\lambda_H + \lambda_L)V_L = \lambda_H V_H$ or equivalently $\frac{\lambda_L}{\lambda_H} = \frac{V_H}{V_L} - 1$.

(2) Suppose $D \in [\widehat{D}, \overline{D}] = [\frac{v_H}{\pi_H}, \frac{v_L}{\pi_L}]$. Using Lemma 1, the L-types may or may not steal while the H-types would never steal. Note that $0 \leq \frac{\pi_L}{r}D < V_L < V_H \leq \frac{\pi_H}{r}D$. Consider four regions for the price P.

1. $P \leq \frac{\pi_L}{r}D$. Both types buy and firm profits are $(\lambda_H + \lambda_L)P$. The price that maximizes profits is $P = \frac{\pi_L}{r}D = \frac{\theta_L}{1-\theta_L}D$. Firm profits are $(\lambda_H + \lambda_L)\frac{\theta_L}{1-\theta_L}D$.
2. $P \in \left(\frac{\pi_L}{r}D, V_L\right]$. The H-types buy and the L-types steal and purchase after being caught. Profits are $\lambda_H P + \lambda_L\theta_L(D+P)$. The price that maximizes profits is $P = V_L$. Firm profits are $\lambda_H V_L + \lambda_L\theta_L(D + V_L)$.
3. $P \in (V_L, V_H]$. The H-types buy and the L-types steal and do not purchase after being caught. Profits are $\lambda_H P + \lambda_L\theta_L D$. The price that maximizes profits is $P = V_H$. Firm profits are $\lambda_H V_H + \lambda_L\theta_L D$.

4. $P > V_H$. The H-types do not consume the product. The L-types steal and do not buy after being caught. Firm profits are $\lambda_L \theta_L D$.

Note that $P = V_L$ (region 2) generates higher profit than $P = \frac{\pi_L}{r} D$ (region 1) given $V_L > \frac{\pi_L}{r} D = \frac{\theta_L}{1-\theta_L} D$. Note also that $P = V_H$ (region 3) generates higher profits than $P > V_H$ (region 4). So the profit-maximizing price is either $P = V_L$ (region 2) or $P = V_H$ (region 3). The profits are equal when

$$\frac{\lambda_L}{\lambda_H} = \frac{1}{\theta_L}\left(\frac{V_H}{V_L} - 1\right).$$

When $\frac{\lambda_L}{\lambda_H}$ is above this threshold, then $P = V_L$; when $\frac{\lambda_L}{\lambda_H}$ is below this threshold, then $P = V_H$.

(3) Suppose $D \in [\underline{D}, \widehat{D}) = [\frac{v_L}{\pi_H}, \frac{v_H}{\pi_H})$. Using Lemma 1, both types may steal. Note that $0 < \frac{\pi_L}{r} D \leq V_L \leq \frac{\pi_H}{r} D < V_H$. Consider five regions for the price.

1. $P \in (0, \frac{\pi_L}{r} D]$. Both types buy rather than steal if $V_i - P > V_i - \theta_i(D + P)$ or equivalently $P < \frac{\theta_i}{1-\theta_i} D = \frac{\pi_i}{r} D$. This is assured by $P \leq \frac{\pi_L}{r} D < \frac{\pi_H}{r} D$. Firm profits are $(\lambda_H + \lambda_L)P$. The price that maximizes profits is $P = \frac{\pi_L}{r} D = \frac{\theta_L}{1-\theta_L} D$. Firm profits are $(\lambda_H + \lambda_L)\frac{\theta_L}{1-\theta_L} D$.

2. $P \in (\frac{\pi_L}{r} D, V_L]$. The H-types buy and L-types steal and purchase after being caught. Firm profits are $\lambda_H P + \lambda_L \theta_L (D + P)$. The price that maximizes profits is $P = V_L$. Firm profits are $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$.

3. $P \in \big(V_L, \frac{\pi_H}{r} D\big]$. The H-types buy and L-types steal and do not purchase after being caught. Profits are $\lambda_H P + \lambda_L \theta_L D$. The price that maximizes profits is $P = \frac{\pi_H}{r} D = \frac{\theta_H}{1-\theta_H} D$. Firm profits are $\lambda_H \frac{\theta_H}{1-\theta_H} D + \lambda_L \theta_L D$.

4. $P \in \big(\frac{\pi_H}{r} D, V_H\big]$. Both types steal; the H-types purchase after being caught and the L-types do not. Profits are $\lambda_H \theta_H (D + P) + \lambda_L \theta_L D$. The price that maximizes profits is $P = V_H$. Firm profits are $\lambda_H \theta_H (D + V_H) + \lambda_L \theta_L D$.

5. $P > V_H$. Both types steal and neither type purchases after being caught. Firm profits are $\lambda_H \theta_H D + \lambda_L \theta_L D$.

Since $V_L \geq \frac{\pi_L}{r} D = \frac{\theta_L}{1-\theta_L} D$, one can show that $P = V_L$ (region 2) generates weakly higher profit than $P = \frac{\pi_L}{r} D$ (region 1). Because $V_H > \frac{\pi_H}{r} D = \frac{\theta_H}{1-\theta_H} D$, $P = \frac{\pi_H}{r} D$ (region 3) is dominated by $P = V_H$ (region 4). Also, $P = V_H$ (region 4) generates higher profits than $P > V_H$ (region 5). So the profit-maximizing price is either $P = V_L$ (region 2) or $P = V_H$

(region 4). The profits are equal when

$$\frac{\lambda_L}{\lambda_H} = \frac{1}{\theta_L}\left[\frac{\theta_H(D+V_H)}{V_L} - 1\right].$$

(4) Suppose $D < \underline{D} = \frac{v_L}{\pi_H}$. Note that $0 \leq \frac{\pi_L}{r}D < \frac{\pi_H}{r}D \leq V_L < V_H$. We consider five regions for the price P.

1. $P \in [0, \frac{\pi_L}{r}D]$. Both types buy and firm profits are $(\lambda_H + \lambda_L)P$. The price that maximizes profits is $P = \frac{\pi_L}{r}D = \frac{\theta_L}{1-\theta_L}D$. Firm profits are $(\lambda_H + \lambda_L)\frac{\theta_L}{1-\theta_L}D$.

2. $P \in (\frac{\pi_L}{r}D, \frac{\pi_H}{r}D]$. The H-types buy and the L-types steal and purchase after being caught. Profits are $\lambda_H P + \lambda_L\theta_L(D+P)$. The price that maximizes profits is $P = \frac{\pi_H}{r}D = \frac{\theta_H}{1-\theta_H}D$. Firm profits are $\lambda_H\frac{\theta_H}{1-\theta_H}D + \lambda_L\theta_L(D + \frac{\theta_H}{1-\theta_H}D) = \frac{\lambda_H\theta_H+\lambda_L\theta_L}{1-\theta_H}D$.

3. $P \in (\frac{\pi_H}{r}D, V_L]$. Both types steal and purchase after being caught. Profits are $(\lambda_H\theta_H + \lambda_L\theta_L)(D+P)$. The price that maximizes profits is $P = V_L$. Firm profits are $(\lambda_H\theta_H + \lambda_L\theta_L)(D+V_L)$.

4. $P \in (V_L, V_H]$. Both types steal; the H-types purchase after being caught and the L-types do not. Profits are $\lambda_H\theta_H(D+P) + \lambda_L\theta_L D$. The price that maximizes profits is $P = V_H$. Firm profits are $(\lambda_H\theta_H + \lambda_L\theta_L)D + \lambda_H\theta_H V_H$.

5. $P > V_H$. Both types steal and neither type purchases after being caught. Firm profits are $\lambda_H\theta_H D + \lambda_L\theta_L D$.

Price $P = V_L$ (region 3) gives higher profits than $P = \frac{\pi_L}{r}D$ (region 1) or $P = \frac{\pi_H}{r}D$ (region 2). Price $P = V_H$ (region 4) gives higher profits than $P > V_H$ (region 5). So the profit-maximizing price is either $P = V_L$ (region 3) or $P = V_H$ (region 4). The profits are equal when

$$\frac{\lambda_L}{\lambda_H} = \frac{\theta_H}{\theta_L}\left[\frac{V_H}{V_L} - 1\right].$$

Proof of Proposition 1. Since $D > \overline{D} = \frac{v_L}{\pi_L}$, Lemma 1 implies that neither type steals if there is no settlement. We first construct the most profitable policy under which the H-types buy and L-types steal the product. Then we show that such a policy generates higher profits than policies under which both types steal or neither type steals.

(1) Suppose that the H-types buy and L-types steal the product. Then the optimal policy must have the property $P \in (V_L, V_H]$. First, if $P > V_H$, then the high types would never

purchase the product. If $P \leq V_L$ then the firm's profits are (weakly) lower than $(\lambda_L + \lambda_H)V_L$. Let $M(T) \equiv \left(\frac{\theta_H}{1-\theta_H e^{-rT}}\right)\left(\frac{1-\theta_L e^{-rT}}{\theta_L}\right)$. As shown in the text, we can rewrite the program as:

$$\text{Max}_{\{P,T\}} \lambda_H P + \lambda_L \left(1 - \theta_L e^{-rT}\right) V_L$$

subject to

$$P \in (V_L, \min\{M(T)V_L, V_H\}]. \tag{23}$$

We consider two cases.

1. $\frac{V_H}{V_L} \leq \frac{\theta_H}{\theta_L}$. For any $T \geq 0$, $M(T)V_L \geq V_H$. Condition (23) becomes $P \in (V_L, V_H]$. Since firm profits increase in P and T, the firm chooses $P = V_H$, $T = \infty$, and $X = \frac{V_L}{\theta_L} = \overline{D} + V_L$ given $\overline{D} = v_L/\pi_L = \frac{1-\theta_L}{\theta_L} V_L$. Firm profits become $\lambda_H V_H + \lambda_L V_L$, which is strictly larger than $\max\{(\lambda_L + \lambda_H)V_L, \lambda_H V_H\}$ (i.e. firm profits if there is no settlement).

2. $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$. Note that $M(0)V_L = \frac{\pi_H}{\pi_L} V_L \geq V_H$ and $M(\infty)V_L = \frac{\theta_H}{\theta_L} V_L < V_H$. Then there exists a unique $\widetilde{T} \in [0, \infty)$ such that $M(\widetilde{T})V_L = V_H$. It can be verified that $e^{-r\widetilde{T}} \equiv \frac{(V_H/\theta_H)-(V_L/\theta_L)}{V_H - V_L}$. If the firm chooses $T \leq \widetilde{T}$, then condition (23) becomes $P \in (V_L, V_H]$, so that the profits are highest when $P = V_H$ and $T = \widetilde{T}$. If the firm chooses $T \geq \widetilde{T}$, then the optimal price should be $P = M(T)V_L \leq V_H$ and firm profits become

$$S^L(T) = \lambda_H \left(\frac{\theta_H}{1 - \theta_H e^{-rT}}\right)\left(\frac{1 - \theta_L e^{-rT}}{\theta_L}\right) V_L + \lambda_L \left(1 - \theta_L e^{-rT}\right) V_L,$$

   which is convex in T. Therefore, if $T \geq \widetilde{T}$, $S^L(T)$ is maximized at either $T = \widetilde{T}$ or $T = \infty$. If $T = \infty$, then $P = \frac{\theta_H}{\theta_L} V_L = \theta_H(\overline{D} + V_L)$ and $X = \frac{V_L}{\theta_L} = \overline{D} + V_L$, with firm profits as $S^L(\infty) = \lambda_H \frac{\theta_H}{\theta_L} V_L + \lambda_L V_L$. If $T = \widetilde{T}$, then $P = V_H$ and $X = \left(\frac{1-\theta_L e^{-r\widetilde{T}}}{\theta_L}\right) V_L = \overline{D} + (1 - e^{-r\widetilde{T}})V_L$, with firm profits as $S^L(\widetilde{T}) = \lambda_H V_H + \lambda_L \left(1 - \theta_L e^{-r\widetilde{T}}\right) V_L$. If and only if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$, we have

$$S^L(\widetilde{T}) - S^L(\infty) = V_L \left\{\lambda_H \frac{V_H}{V_L} + \lambda_L \left(1 - \theta_L \frac{(V_H/\theta_H) - (V_L/\theta_L)}{V_H - V_L}\right)\right\} - V_L \left(\lambda_H \frac{\theta_H}{\theta_L} + \lambda_L\right) > 0.$$

   Therefore, the firm chooses $T = \widetilde{T}$ if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ and $T = \infty$ if otherwise.

Moreover, note that

$$S^L(\infty) = \left(\lambda_H \frac{\theta_H}{\theta_L} + \lambda_L\right) V_L > (\lambda_H + \lambda_L)V_L,$$

and,

$$S^L(\widetilde{T}) > \lambda_H V_H.$$

Therefore, by offering the above settlement policies, the firm can generate higher profits than in the no-settlement benchmark.

(2) It remains to show that the firm would not implement a policy under which both types steal. Suppose to the contrary that the firm implements a policy $\{P, X, T\}$ under which both types steal. Then we must have $P \in (V_L, V_H]$. To see this, note that if $P \leq V_L$ then firm profits are (weakly) lower than $(\lambda_L + \lambda_H)V_L$. If $P > V_H$, any consumer who is caught stealing will settle the case and stop consuming the product after finishing the license of duration T, and firm profits are $\lambda_H \theta_H X + \lambda_L \theta_L X$. But the firm could generate at least the same profits by offering $P' = \theta_H X \leq V_H$, $X' = X$ and $T' = T$, under which the H-types would purchase while the L-type may purchase or steal the product. By assumption, if the firm is indifferent between purchasing and stealing by consumers, it adopts the policy where the consumers purchase the product.

Now suppose $P \in (V_L, V_H]$ and both types steal. The firm's program is

$$\text{Max}_{\{P \in (V_L, V_H], X, T\}} \quad \lambda_H \theta_H (X + e^{-rT} P) + \lambda_L \theta_L X$$

subject to

$$V_H - \theta_H (X + e^{-rT} P) > V_H - P$$

$$(1 - \theta_L e^{-rT}) V_L - \theta_L X \geq 0$$

The first condition is the H-type's incentive-compatibility constraint. This constraint has a strict inequality, as otherwise the H-types will buy. The second condition is the L-type's individual-rationality constraint, which must bind under the optimal policy. Then we can re-write the firm's program as

$$\text{Max}_{\{P \in (V_L, V_H], T\}} \lambda_H \theta_H \left[\frac{V_L}{\theta_L} + e^{-rT}(P - V_L)\right] + \lambda_L (1 - \theta_L e^{-rT}) V_L.$$

subject to

$$V_H - \theta_H \left(\frac{V_L}{\theta_L} + e^{-rT}(P - V_L)\right) > V_H - P \tag{24}$$

The objective function increases in P and is monotonic in T, so the optimal price is $P = V_H$ and firm profits are either maximized by $T = 0$ or $T = \infty$. If $T = 0$, then condition (24) implies $V_H > \frac{\pi_H}{\pi_L} V_L$, a contradiction to Assumption 1. If $T = \infty$, then firm profits are $\lambda_H \frac{\theta_H}{\theta_L} V_L + \lambda_L V_L$. However, as shown in case (1), the firm can generate the same profits by choosing $P' = \frac{\theta_H}{\theta_L} V_L = \theta_H(\overline{D} + V_L)$, $X' = \frac{V_L}{\theta_L} = \overline{D} + V_L$, and $T' = \infty$, under which the H-types buy and L-types steal. By assumption, the firm chooses this alternative policy under which only the L-types steal the product.

Proof of Proposition 2. Suppose $D \in \left[\widehat{D}, \overline{D}\right] = \left[\frac{v_H}{\pi_H}, \frac{v_L}{\pi_L}\right]$. Since $D \leq \frac{v_L}{\pi_L}$, Lemma 1 implies that, if settlement is not available, the L-types either steal or buy the product. If the firm offers the settlement policy $\{X = D, T = 0\}$, profits are the same as in the no-settlement benchmark. The firm prefers settlement to no-settlement. In the following, We first construct the most profitable policy under which the H-types buy and L-types steal the product. Then we show that the firm would not implement policies under which both types steal.

(1) Suppose that the H-types buy and L-types steal the product. Then the optimal policy must have the property $P \in [V_L, V_H]$. First, if $P > V_H$, then the high types would never purchase the product. If $P < V_L$ then the firm's profits are lower than $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$, which can be generated under the policy $\{P = V_L, X = D, T = 0\}$ (Lemma 1 implies that the H-types purchase and the L-types steal the product under this policy). Note that, if $P = V_L$, the L-types will purchase the product after being caught and therefore the settlement contract $\{X = D, T = 0\}$ maximizes firm profits.

Now consider the region $P \in (V_L, V_H]$. The firm's program is

$$\text{Max}_{\{P \in (V_L, V_H], X, T\}} \quad \lambda_H P + \lambda_L \theta_L X$$

subject to

$$V_H - P \geq V_H - \theta_H (X + e^{-rT} P)$$

$$X \leq D + (1 - e^{-rT}) V_L.$$

The first condition is the H-type's ex-ante incentive-compatibility constraint. The second condition is the L-type's ex-post incentive-compatibility constraint to settle the lawsuit, which must bind under the optimal policy. So, the optimal settlement is $X = D + (1 - e^{-rT}) V_L$. Then we can rewrite the H-type's incentive-compatibility constraint as

$$P \leq \widetilde{M}(T) \equiv \frac{\theta_H}{1 - \theta_H e^{-rT}} [D + (1 - e^{-rT}) V_L].$$

Given $D \geq \frac{v_H}{\pi_H}$, it can be shown that $\widetilde{M}(T)$ strictly decreases in T with $\widetilde{M}(\infty) = \theta_H (D + V_L)$

and $\widetilde{M}(0) = \frac{\theta_H}{1-\theta_H}D$. The firm's program can be re-written as

$$\text{Max}_{\{P,T\}} \quad \lambda_H P + \lambda_L \theta_L [D + (1 - e^{-rT})V_L]$$

subject to

$$P \in (V_L, \min\{V_H, \widetilde{M}(T)\}]. \tag{25}$$

We consider two cases.

1. $V_H \leq \theta_H(D + V_L)$. For any $T \geq 0$, $\widetilde{M}(T) \geq V_H$. Condition (25) becomes $P \in (V_L, V_H]$. Since firm profits increase in P and T, the firm chooses $P = V_H$, $T = \infty$, and $X = D + V_L$. Firm profits become $\lambda_H V_H + \lambda_L \theta_L (D + V_L)$, higher than the profits under the policy $\{P = V_L, X = D, T = 0\}$.

2. $V_H > \theta_H(D + V_L)$. Since $D \geq \frac{v_H}{\pi_H} = \frac{1-\theta_H}{\theta_H} V_H > \frac{1-\theta_H}{\theta_H} V_L$, we have $\widetilde{M}(0) = \frac{\theta_H}{1-\theta_H} D \geq V_H$ and $\widetilde{M}(\infty) = \theta_H(D + V_L) \in (V_L, V_H)$. Therefore, there exists a unique $\widehat{T} \in [0, \infty)$ such that $V_H \equiv \widetilde{M}(\widehat{T})$. If the firm chooses $T \leq \widehat{T}$, then condition (25) becomes $P \in (V_L, V_H]$, so that the profits are highest when $P = V_H$ and $T = \widehat{T}$. If the firm chooses $T \geq \widetilde{T}$, then the optimal price should be $P = \widetilde{M}(T) \leq V_H$ and firm profits become

$$S^{LD}(T) = \lambda_H \frac{\theta_H}{1 - \theta_H e^{-rT}} [D + (1 - e^{-rT})V_L] + \lambda_L \theta_L [D + (1 - e^{-rT})V_L],$$

which can be shown to be convex in T. Accordingly, firm profits are maximized by either $T = \widehat{T}$ or $T = \infty$. It can be verified that both $S^{LD}(\widehat{T})$ and $S^{LD}(\infty)$ are higher than $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$, that is, the profits under the policy $\{P = V_L, X = D, T = 0\}$. Note that

$$S^{LD}(\widehat{T}) - S^{LD}(\infty) = [V_H - \theta_H(D + V_L)] \left[\lambda_H - \lambda_L \frac{\theta_L V_L}{\theta_H (V_H - V_L)} \right],$$

which is positive if and only if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L} \left(\frac{V_H}{V_L} - 1 \right)$. Therefore, the firm chooses $T = \widehat{T}$ if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L} \left(\frac{V_H}{V_L} - 1 \right)$ and $T = \infty$ if otherwise.

(2) It remains to show that the firm would not implement a policy under which both types steal. Suppose to the contrary that the firm implements a policy $\{P, X, T\}$ under which both types steal. Consider five cases. In each case, we identify alternative policies under which the firm generates weakly higher profits and induces only the L-types to steal.

1. If $P \leq V_L$, then firm profits are weakly lower than $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$, which are the profit when the firm chooses $\{P = V_L,\ X = D, T = 0\}$ and only the L-types steal.

2. If $P > V_H$ and both types settle lawsuits after being caught, firm profits are $(\lambda_H\theta_H + \lambda_L\theta_L)X$, which is independent of T. A long duration T increases consumers' incentives to steal and settle, thereby allowing the firm to raise X. If $T = \infty$ is feasible, since the H-types steal, we have $V_H - \theta_H X \geq 0$; since the L-types settle lawsuits, we have $X \leq D + V_L$. Therefore, the optimal settlement amount X is weakly lower than $\min\left\{\frac{V_H}{\theta_H}, D + V_L\right\}$ and firm profits are weakly lower than $(\lambda_H\theta_H + \lambda_L\theta_L)\min\left\{\frac{V_H}{\theta_H}, D + V_L\right\}$. However, as shown in part (1), the firm can generate weakly higher profits $\lambda_H \min\{V_H, \theta_H(D + V_L)\} + \lambda_L\theta_L(D + V_L)$ by using $\{P = \min\{V_H, \theta_H(D + V_L)\},\ X = D + V_L, T = \infty\}$, under which only the L-types steal.

3. If $P > V_H$ and only the H-types settle lawsuits after being caught, firm profits are $\lambda_H\theta_H X + \lambda_L\theta_L D$. If $T = \infty$ is feasible, since the H-types settle lawsuits, we have $X \leq D + V_H$. Therefore, the optimal settlement X is weakly lower than $D + V_H$ and firm profits are weakly lower than $\lambda_H\theta_H(D + V_H) + \lambda_L\theta_L D$. It is not difficult to verify that $\lambda_H\theta_H(D + V_H) + \lambda_L\theta_L D$ is lower than $S^{LD}(\widehat{T})$ if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ and lower than $S^{LD}(\infty)$ if otherwise. As shown in part (1), the firm can generate $\min\{S^{LD}(\widehat{T}), S^{LD}(\infty)\}$ by using policies under which only the L-types steal.

4. If $P \in (V_L, V_H]$ and both types settle lawsuits after being caught, firm profits are $\lambda_H\theta_H(X + e^{-rT}P) + \lambda_L\theta_L X$. It is easy to verify that, under the optimal policy, the L-types are indifferent between settlement and litigation, which implies $X = D + (1 - e^{-rT})V_L$. Then firm profits become $\lambda_H\theta_H[D + (1 - e^{-rT})V_L + e^{-rT}P] + \lambda_L\theta_L[D + (1 - e^{-rT})V_L]$, which is monotonic in T. If $T = 0$, firm profits are weakly lower than $\lambda_H\theta_H[D + V_H] + \lambda_L\theta_L D$, while the firm can generate higher profits $\min\{S^{LD}(\widehat{T}), S^{LD}(\infty)\}$ by using policies under which only the L-types steal. If $T = \infty$, firm profits are $\lambda_H\theta_H(D + V_L) + \lambda_L\theta_L(D + V_L)$, while as shown in part (1) the firm can generate the same profits by using the policy $\{P = \theta_H(D + V_L),\ X = D + V_L, T = \infty\}$, under which only the L-types steal.

5. If $P \in (V_L, V_H]$ and only the H-types settle lawsuits after being caught, firm profits are $\lambda_H\theta_H(X + e^{-rT}P) + \lambda_L\theta_L D$. Under the optimal policy, the H-types are indifferent between settlement and litigation, which implies $X = D + (1 - e^{-rT})P$. Therefore, firm profits are weakly lower than $\lambda_H\theta_H(D + V_H) + \lambda_L\theta_L D$. However, as shown in part (1), the firm can generate higher profits $\min\{S^{LD}(\widehat{T}), S^{LD}(\infty)\}$ by using policies under which only the L-types steal.

Proof of Proposition 3. Suppose $D \in \left[\underline{D}, \widehat{D}\right) = \left[\frac{v_L}{\pi_H}, \frac{v_H}{\pi_H}\right)$. If the firm offers the settlement policy $\{X = D, T = 0\}$, profits are the same as in the no-settlement benchmark. We first compare the most profitable policy under which only the L-types steal to the most profitable policy under which both types steal and settle. Then we show that the firm would not implement policies under which both types steal but only the H-types settle.

(1) Suppose that the H-types buy and L-types steal the product. Similar to the proof of Proposition 2, the optimal policy must have the property $P \in [V_L, V_H]$. If $P = V_L$, the settlement contract $\{X = D, T = 0\}$ is optimal and firm profits are $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$.

Now consider the region $P \in (V_L, V_H]$. Similar to the proof of Proposition 2, the firm's program can be rewritten as

$$\text{Max}_{\{P,T\}} \quad \lambda_H P + \lambda_L \theta_L [D + (1 - e^{-rT})V_L]$$

subject to

$$P \in (V_L, \min\{V_H, \widetilde{M}(T)\}],$$

where $\widetilde{M}(T) \equiv \frac{\theta_H}{1-\theta_H e^{-rT}}[D + (1 - e^{-rT})V_L]$ strictly decreases in T. Since $D \in \left[\frac{v_L}{\pi_H}, \frac{v_H}{\pi_H}\right)$, we have $V_L \le \widetilde{M}(\infty) \le \widetilde{M}(T) \le \widetilde{M}(0) < V_H$ for any $T \ge 0$. Therefore, the optimal price is $P = \widetilde{M}(T)$. Firm profits become $S^{LD}(T)$ which is convex in T as shown in the proof of Proposition 2. Moreover, it can be verified that $S^{LD}(\infty)$ is weakly larger than $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$, that is, the profits under the policy $\{P = V_L, X = D, T = 0\}$. Accordingly, profits are maximized by either $T = 0$ or $T = \infty$. Note that $S^{LD}(0) = \lambda_H \frac{\theta_H}{1-\theta_H} D + \lambda_L \theta_L D$ and $S^{LD}(\infty) = \lambda_H \theta_H (D + V_L) + \lambda_L \theta_L (D + V_L)$. It is easy to verify that $S^{LD}(0) - S^{LD}(\infty) > 0$ if and only if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{\theta_H}{1-\theta_H}\frac{D}{V_L} - 1\right)$.

(2) Now suppose that both types steal the product and settle lawsuits. If $P \le V_L$, then firm profits are weakly lower than $\lambda_H V_L + \lambda_L \theta_L (D + V_L)$, which are the profit when the firm chooses $\{P = V_L,\ X = D, T = 0\}$ and only the L-types steal (see the proof of Lemma 3). Therefore, the optimal policy must have the property $P > V_L$. Consider two regions of the price.

1. If $P > V_H$, firm profits are $(\lambda_H \theta_H + \lambda_L \theta_L) X$, which is independent of T. A long duration T increases consumers' incentives to steal and settle, thereby allowing the firm to raise X. If $T = \infty$ is feasible, since the L-types settle lawsuits, we have $X \le D + V_L$. Therefore, firm profits are weakly lower than $(\lambda_H \theta_H + \lambda_L \theta_L)(D + V_L) = S^{LD}(\infty)$. However, as shown in part (1), the firm can generate profits $S^{LD}(\infty)$ by using $\{P = \theta_H (D + V_L),\ X = D + V_L, T = \infty\}$, under which only the L-types steal. Therefore, the firm would not choose $P > V_H$ under which both types steal.

2. If $P \in (V_L, V_H]$, the firm's program is

$$\text{Max}_{\{P \in (V_L, V_H], X, T\}} \quad \lambda_H \theta_H (X + e^{-rT} P) + \lambda_L \theta_L X$$

subject to

$$P > \theta_H (X + e^{-rT} P) \tag{26}$$

$$X \leq D + (1 - e^{-rT}) V_L \tag{27}$$

Under the optimal policy, condition (27) must bind. So the optimal settlement amount is $X = D + (1 - e^{-rT})V_L$. The optimal price is $P = V_H$. Since $D < \frac{v_H}{\pi_H} = \frac{1-\theta_H}{\theta_H} V_H$, condition (26) holds for any $T \geq 0$. Therefore, firm profits can be rewritten as

$$(\lambda_H \theta_H + \lambda_L \theta_L)[D + (1 - e^{-rT})V_L] + \lambda_H \theta_H e^{-rT} V_H,$$

which is maximized by either $T = 0$ or $T = \infty$. If $T = \infty$, firm profits are $\lambda_H \theta_H (D + V_L) + \lambda_L \theta_L (D + V_L)$. However, as shown in part (1), the firm can generate the same profits by using $\{P = \theta_H (D + V_L),\ X = D + V_L, T = \infty\}$, under which only the L-types steal. If $T = 0$, firm profits are $\lambda_H \theta_H (D + V_H) + \lambda_L \theta_L D$, which is higher than $S^{LD}(0) = \lambda_H \frac{\theta_H}{1-\theta_H} D + \lambda_L \theta_L D$ given $D < \frac{v_H}{\pi_H} = \frac{1-\theta_H}{\theta_H} V_H$.

Summarizing the analysis in parts (1) and (2), the firm chooses either $\{P = \theta_H (D + V_L), X = D + V_L,\ T = \infty\}$ with profits $S^{LD}(\infty)$ or $\{P = V_H, X = D,\ T = 0\}$ with profits $\lambda_H \theta_H (D + V_H) + \lambda_L \theta_L D$. The profits under the two policies are the same if $\frac{\lambda_L}{\lambda_H} = \frac{\theta_H}{\theta_L} \left(\frac{V_H}{V_L} - 1 \right)$.

(3) It remains to show that the firm would not implement a policy under which both types steal the product but only the H-types settle lawsuits. Suppose to the contrary that the firm implements a policy $\{P, X, T\}$ under which both types steal but only the H-types settle.

1. If $P \leq V_L$, the H-types accepts settlement when $X \leq D + (1 - e^{-rT})P$, which however implies that the L-types will accept settlement as well.

2. If $P > V_H$, firm profits are $\lambda_H \theta_H X + \lambda_L \theta_L D$. A longer duration T increases the H-type's incentives to steal and settle, thereby allowing the firm to raise X. If $T = \infty$ is feasible, since the H-types settle lawsuits, we have $X \leq D + V_H$. Therefore, firm profits are weakly lower than $\lambda_H \theta_H (D + V_H) + \lambda_L \theta_L D$. However, as shown in part (2), the firm can generate profits of $\lambda_H \theta_H (D + V_H) + \lambda_L \theta_L D$ by using $\{P = V_H, X = D,\ T = 0\}$, under which both types settle lawsuits. By assumption, the firm adopts this alternative policy under which the L-types settle lawsuits instead of going to court.

3. If $P \in (V_L, V_H]$, firm profits are $\lambda_H\theta_H(X + e^{-rT}P) + \lambda_L\theta_L D$. Under the optimal policy, the H-types are indifferent between settlement and litigation, which implies $X = D + (1 - e^{-rT})P$. Therefore, firm profits are weakly lower than $\lambda_H\theta_H(D+V_H)+\lambda_L\theta_L D$. Again the firm prefers the alternative policy $\{P = V_H, X = D, T = 0\}$, under which both types settle lawsuits.

Proof of Proposition 4. Suppose $D < \underline{D} = \frac{v_L}{\pi_H}$. If the firm offers the settlement policy $\{X = D, T = 0\}$, profits are the same as in the no-settlement benchmark. Lemma 3 implies that, under this policy, both types steal the product and firm profits are

$$\max\{\lambda_H\theta_H(D + V_H) + \lambda_L\theta_L D, (\lambda_H\theta_H + \lambda_L\theta_L)(D + V_L)\}.$$

Let $T \geq 0$. The firm would never implement a policy under which the H-types buy and L-types steal the product. To see this, suppose that the H-types buy and the L-types steal. If $P < V_L$, then the firm's profits are lower than $\lambda_H V_L + \lambda_L\theta_L(D + V_L)$, while the firm can generate higher profits $(\lambda_H\theta_H+\lambda_L\theta_L)(D+V_L)$ by using the policy $\{P = V_L, X = D, T = 0\}$. If $P \geq V_L$, the H-types would steal because $P \geq V_L > \frac{\pi_H}{r}D = \frac{\theta_H}{1-\theta_H}D$ or equivalently $P > \theta_H(D + P)$. Therefore, we can restrict to policies under which both types steal the product.

Using the same analysis as in the proof of Proposition 3, one can show that the firm would not implement policies under which both types steal but only the H-types settle lawsuits.

Suppose that both types steal and settle. Consider three regions of the price.

1. If $P \leq V_L$, the profit-maximizing price must be $P = V_L$. To see this, note that, if $P < V_L$, firm profits are lower than $\lambda_H V_L + \lambda_L\theta_L(D + V_L)$. However, since $D < \frac{v_L}{\pi_H} = \frac{1-\theta_H}{\theta_H}V_L$, the firm can generate high profits $(\lambda_H\theta_H + \lambda_L\theta_L)(D + V_L)$ by using $\{P = V_L,\ X = D, T = 0\}$. In fact, there exist a continuum of policies with $P = V_L$ and $X = D + (1 - e^{-rT})V_L$ *for any* $T \geq 0$ which generate the same profits. Without loss of generality, we assume that the firm choose the one with the longest duration, that is, $\{P = V_L, X = D + V_L, T = \infty\}$.

2. If $P > V_H$, firm profits are $(\lambda_H\theta_H+\lambda_L\theta_L)X$, which is independent of T. A long duration T increases consumers' incentives to steal and settle, thereby allowing the firm to raise X. If $T = \infty$, since the L-types settle lawsuits, we have $X \leq D + V_L$. If $X = D + V_L$, then the H-types have incentives to steal the product because $V_H > V_L > \theta_H(D+V_L)$. Therefore, in this region, the optimal policy is $\{P > V_H, X = D + V_L, T = \infty\}$, with firm profits as $(\lambda_H\theta_H + \lambda_L\theta_L)(D + V_L)$.

3. If $P \in (V_L, V_H]$, firm profits are $\lambda_H\theta_H(X + e^{-rT}P) + \lambda_L\theta_L X$. Similar to the proof of Proposition 3, the optimal settlement amount is $X = D+(1-e^{-rT})V_L$ and firm profits

can be rewritten as

$$(\lambda_H\theta_H + \lambda_L\theta_L)[D + (1 - e^{-rT})V_L] + \lambda_H\theta_H e^{-rT} V_H,$$

which is maximized by either $T = 0$ or $T = \infty$. If $T = \infty$, the optimal settlement is $X = D + V_L$ and the price satisfies $P \in (V_L, V_H]$, with profits as $\lambda_H\theta_H(D + V_L) + \lambda_L\theta_L(D + V_L)$. If $T = 0$, the optimal settlement is $X = D$ and the optimal price is $P = V_H$, with profits as $\lambda_H\theta_H(D + V_H) + \lambda_L\theta_L D$.

Summarizing the three cases, the firm can generate profits of $\lambda_H\theta_H(D+V_L)+\lambda_L\theta_L(D+V_L)$ by using $\{P \geq V_L, X = D + V_L, T = \infty\}$ or profits of $\lambda_H\theta_H(D + V_H) + \lambda_L\theta_L D$ by using $\{P = V_H, X = D, T = 0\}$. The profits are the same if $\frac{\lambda_L}{\lambda_H} = \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$.

Proof of Corollary 2. Comparing Lemma 3 to Propositions 2, 3, and 4, we can see that settlement with licensing strictly increases social welfare if $D \in \left[\widehat{D}, \overline{D}\right]$ and $\frac{\lambda_L}{\lambda_H} < \frac{1}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ or if $D \in \left[\underline{D}, \widehat{D}\right)$ and $\frac{\lambda_L}{\lambda_H} \in \left(\frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right), \frac{1}{\theta_L}\left[\frac{\theta_H(D+V_H)}{V_L} - 1\right]\right)$, and has no effect otherwise.

Proof of Proposition 5. We start by characterizing the firm's optimal settlement offer after catching a stealing consumer. First, suppose that the H-types buy and the L-types steal the product. Once a stealing consumer is caught, the firm has the correct belief that the consumer is of the L-type. Note that the stealing consumer has the option of purchasing the product at the market price P. Therefore, the firm's optimal settlement offer is $\{X = D + \min\{V_L, P\}, T = \infty\}$.

Now suppose that both types steal the product. Once a stealing consumer is caught, the firm has to update its belief about the consumer's type. Specifically, the posterior probability for the stealing consumer to be of the H-type is

$$\frac{\lambda_H\theta_H}{\lambda_H\theta_H + \lambda_L\theta_L}.$$

If $P > V_H$, then the optimal settlement offer is either $\{X = D + V_H, T = \infty\}$, under which the H-types settle while the L-types go to litigation, or $\{X = D + V_L, T = \infty\}$, under which both types settle lawsuits. The firm prefers $X = D + V_H$ and $T = \infty$ if and only if

$$\frac{\lambda_H\theta_H}{\lambda_H\theta_H + \lambda_L\theta_L}(D + V_H) + \left(1 - \frac{\lambda_H\theta_H}{\lambda_H\theta_H + \lambda_L\theta_L}\right) D > D + V_L, \tag{28}$$

or equivalently,

$$\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right). \tag{29}$$

If $P \in (V_L, V_H]$, then once caught, the H-types are willing to buy the product at price P even without settlement. Therefore, the firm can generate the same profit by offering either $\{X = D + P, T = \infty\}$, under which the L-types reject the offer, or $\{X = D, T = 0\}$, under which both types accept the offer. By assumption, the firm chooses the policy with less stealing, $\{X = D, T = 0\}$. Thus, if $P \in (V_L, V_H]$, the optimal settlement offer is either $\{X = D, T = 0\}$, under which only the H-types consume the product after settlement, or $\{X = D + V_L, T = \infty\}$, under which both types consume the product after settlement. The firm prefers $X = D$ and $T = 0$ if and only if

$$\frac{\lambda_H \theta_H}{\lambda_H \theta_H + \lambda_L \theta_L}(D + P) + \left(1 - \frac{\lambda_H \theta_H}{\lambda_H \theta_H + \lambda_L \theta_L}\right) D > D + V_L, \tag{30}$$

or equivalently,

$$\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{P}{V_L} - 1\right). \tag{31}$$

If $P \leq V_L$, then any stealing consumer who rejects the settlement offer will purchase the product in the market. Therefore, there exist a continuum and equivalent settlement offers with $X = D + (1 - e^{-rT})P$ *for any* $T \geq 0$, under which any stealing consumer will settle lawsuits and buy the product after finishing the license of duration T.

When property rights are strong ($D > \overline{D} = v_L/\pi_L = \left(\frac{1-\theta_L}{\theta_L}\right)V_L$) and the firm has limited commitment power, no consumer would steal the product and the equilibrium is the same as in the no-settlement benchmark. To see this, first suppose to the contrary that the H-types buy and the L-types steal the product. As shown earlier, the firm's optimal settlement offer would be $\{X = D + \min\{V_L, P\}, T = \infty\}$. It is not difficult to see that the optimal price should be $P \geq V_L$. However, in this case, the L-types would not steal since $D > \left(\frac{1-\theta_L}{\theta_L}\right)V_L$, a contradiction. Similarly, suppose that both types would steal the product. Then under the optimal settlement offers characterized earlier, neither type would steal the product, a contradiction. Finally, since no consumer steals when property rights are strong, the highest profit that the firm can generate is the same as in the no-settlement benchmark. So, the results in Lemma 2 can be supported by the off-equilibrium settlement offer $X^* = D$ with $T^* = 0$ and the firm's posterior belief that only H-types would steal.

When property rights are weak, the proof is similar to the proofs of Propositions 2-4 and therefore omitted.

Settling Lawsuits with Pirates: Online Appendix B

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In this online appendix, we first show that, given any general mechanism, there exists a simple policy $\{P, X, T\}$ generating the same profits for the firm. Then we provide the technical details of the extensions discussed in Subsections 4.2 - 4.5.

General Mechanisms

Consider the following mechanism $\{P, (P_1, X_1, T_1), (P_2, X_2, T_2)\}$ where P is the market price and P_1 (or P_2) is the (future) price for any stealing consumer who has chosen the settlement contract (X_1, T_1) (or (X_2, T_2)). Note that, after being caught, stealing consumers can still choose to purchase the product at price P from the market.

We first show that there exists another mechanism $\{P, (X_1', T_1'), (X_2', T_2')\}$ that gives the firm the same profits. Suppose that, after being caught, a consumer chooses a settlement contract (X_i, T_i). If the consumer purchases the product at price P_i after the license of duration T_i expires, then he will always consume the product in the future. Thus, the mechanism (P_i, X_i, T_i) can be replaced by an equivalent contract (X_i', T_i'), where $T_i' = \infty$ and $X_i' = X_i + e^{-rT_i}P$. If the consumer does not purchase the product after the license of duration T_i expires, then the mechanism (P_i, X_i, T_i) can be replaced by (P, X_i, T_i). To summarize, we can restrict our attention to the mechanism $\{P, (X_1, T_1), (X_2, T_2)\}$.

Next, we will show that, given any mechanism $\{P, (X_1, T_1), (X_2, T_2)\}$, there exists another simple policy $\{P', X', T'\}$ that gives the firm weakly higher profits.

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If $P \leq V_L$ or if only the L-types steal the product, obviously the firm can use a simple policy. Now consider a mechanism $\{P > V_L, (X_1, T_1), (X_2, T_2)\}$ under which both types steal the product and, once being caught, the L-types choose the contract (X_1, T_1) and the H-types choose (X_2, T_2). Consider two cases.

Case 1. First, if $P > V_H$, the ex-post incentive-compatibility constraint for the H-types to choose (X_2, T_2) is

$$\int_0^{T_2} v_H e^{-rt} dt - X_2 \geq \int_0^{T_1} v_H e^{-rt} dt - X_1,$$

or equivalently,

$$X_2 + e^{-rT_2} V_H \leq X_1 + e^{-rT_1} V_H.$$

Similarly, the ex-post incentive-compatibility constraint for the L-types to choose (X_1, T_1) is

$$X_1 + e^{-rT_1} V_L \leq X_2 + e^{-rT_2} V_L.$$

Thus, the firm chooses $\{P > V_H, (X_1, T_1), (X_2, T_2)\}$ to maximize its profit subject to the ex-ante individual-rationality constraints and the ex-post incentive-compatibility constraints for both types:

$$\text{Max}_{(P > V_H, (X_1, T_1), (X_2, T_2))} \quad \lambda_H \theta_H X_2 + \lambda_L \theta_L X_1 \tag{1}$$

subject to

$$(1 - \theta_H e^{-rT_2}) V_H - \theta_H X_2 \geq 0 \tag{2}$$

$$(1 - \theta_L e^{-rT_1}) V_L - \theta_L X_1 \geq 0 \tag{3}$$

$$X_2 + e^{-rT_2} V_H \leq X_1 + e^{-rT_1} V_H \tag{4}$$

$$X_1 + e^{-rT_1} V_L \leq X_2 + e^{-rT_2} V_L. \tag{5}$$

We must have $T_2 = \infty$ in the optimal policy. If not, then the firm could raise its profits by choosing an alternative policy $\{P, (X_1, T_1), (X_2', T_2')\}$, where $T_2' = T_2 + \varepsilon$ and $X_2' = X_2 + (e^{-rT_2} - e^{-r(T_2+\varepsilon)}) V_H$ for positive but arbitrarily small ε. Note that $X_2 + e^{-rT_2} V_H = X_2' + e^{-rT_2'} V_H$. Under this alternative policy, conditions (2), (3), and (4) still hold. Condition (5) also holds because

$$X_1 + e^{-rT_1} V_L \leq X_2 + e^{-rT_2} V_L < X_2' + e^{-rT_2'} V_L.$$

Therefore, if $P > V_H$ and both types steal the product, $T_2 = \infty$ in the optimal policy. However, given $T_2 = \infty$, the firm could generate the same profits by choosing another simple policy $\{P' = \theta_H X_2,\ X' = X_1, T' = T_1\}$. To see this, note that, under this alternative policy, the H-types will purchase the product while the L-types will steal it because

$$P' = \theta_H X_2 \geq \theta_H(X_1 + e^{-rT_1}V_L) > \theta_L(X_1 + e^{-rT_1}V_L),$$

where the first inequality follows from $T_2 = \infty$ and condition (5).

Case 2. If $P \in (V_L, V_H]$ and both types steal the product, then the H-types who are caught stealing will settle the lawsuit and purchase the product after the license expires. Thus, the ex-post incentive-compatibility constraint for the H-types to choose (X_2, T_2) is

$$X_2 + e^{-rT_2}P \leq X_1 + e^{-rT_1}P.$$

The firm's program is

$$\text{Max}_{(P\in(V_L,V_H],(X_1,T_1),(X_2,T_2))} \quad \lambda_H\theta_H(X_2 + e^{-rT_2}P) + \lambda_L\theta_L X_1 \tag{6}$$

subject to

$$V_H - \theta_H(X_2 + e^{-rT_2}P) > V_H - P \tag{7}$$

$$(1 - \theta_L e^{-rT_1})V_L - \theta_L X_1 \geq 0 \tag{8}$$

$$X_2 + e^{-rT_2}P \leq X_1 + e^{-rT_1}P \tag{9}$$

$$X_1 + e^{-rT_1}V_L \leq X_2 + e^{-rT_2}V_L. \tag{10}$$

The first condition is the H-types' ex-ante incentive-compatibility constraint, which is not binding as otherwise they would purchase the product. In the optimal policy, the ex-post incentive-compatibility constraint for the H-types to choose (X_2, T_2) must bind, as otherwise the firm could raise profits by increasing X_2 marginally. That is, $X_2 + e^{-rT_2}P = X_1 + e^{-rT_1}P$. Since the firm would get the same profits no matter whether H-types choose (X_2, T_2) or (X_1, T_1), the firm could use a simple policy $\{P' = P,\ X' = X_1,\ T' = T_1\}$.

Section 4.2. Network Effects

Suppose that the number of users at each moment is $N(T)$ and a type-i consumer's discounted gross value from using the product is $N(T)V_i$, for $i = H, L$. For simplicity, we focus on the scenario with strong property rights, $D > \overline{D}$, where $\overline{D} = v_L/\pi_L = \frac{1-\theta_L}{\theta_L}V_L$. Consider policies $\{P, X, T\}$ under which the H-type consumers buy and the L-types steal the product. Note that, at any moment t, the L-types who are still active but were caught in the interval $(-\infty, t-T]$ have stopped using the product.

Lemma B.1. *Suppose that the firm chooses a policy under which the H-types buy and L-types steal the product. The number of users at each moment is* $N(T) = \lambda_H + \lambda_L\big(1 - \frac{\pi_L}{\alpha+\pi_L}e^{-\alpha T}\big)$, *increasing in* T.

Proof of Lemma B.1. Consider any time t and a cohort of consumers who were born at time $y < t$. So, the variable y is a birth cohort. To construct $N(T)$, We first derive the density of birth cohorts y for consumers who are active at time t.

Claim 1. *The density of birth cohorts y for all consumers who are active at time t is* $\phi(y \mid t) = \alpha e^{-\alpha(t-y)}$ *where* $y \in (-\infty, t]$.[1]

Proof of Claim 1. We will prove this claim using the odds form of Bayes' rule. Let t be the event that a consumer is still active at time t, and consider two birth cohorts, $y_i < t, i = 1, 2$. Recall that the probability that a consumer from birth cohort $y_i < t$ has exited the market by time t is $1 - e^{-\alpha(t-y_i)}$. So, the probability that the consumer from the birth cohort y_i is still active at time t is $\Pr(t \mid y_i) = e^{-\alpha(t-y_i)}$. So the likelihood ratio is

$$\frac{\Pr(t \mid y_1)}{\Pr(t \mid y_2)} = \frac{e^{-\alpha(t-y_1)}}{e^{-\alpha(t-y_2)}}.$$

The prior (unconditional) odds ratio of y_1 to y_2 is $\Pr(y_1)/\Pr(y_2) = 1$. Therefore the posterior (conditional) odds ratio is:

$$\frac{\phi(y_1 \mid t)}{\phi(y_2 \mid t)} = \frac{\Pr(t \mid y_1)}{\Pr(t \mid y_2)} \times \frac{\Pr(y_1)}{\Pr(y_2)} = \frac{e^{-\alpha(t-y_1)}}{e^{-\alpha(t-y_2)}}.$$

Taking the natural logarithm of both sides and rearranging,

$$\ln\phi(y_1 \mid t) - \ln\phi(y_2 \mid t) = \alpha(y_1 - y_2).$$

Dividing both sides by $y_1 - y_2$ and taking the limit as $y_1 - y_2$ goes to zero gives

$$\tfrac{d}{dy}\left[\ln\phi(y \mid t)\right] = \alpha.$$

The derivative of the left-hand side is $\phi'(y \mid t)/\phi(y \mid t)$, so we have the first-order differential equation

$$\phi'(y \mid t) - \alpha\phi(y \mid t) = 0.$$

[1] Note that $\phi(y \mid t)$ is an increasing function of y. Younger generations of consumers are more heavily represented in the consumer population at time t. Furthermore, $\phi(y \mid t)$ approaches zero at y approaches $-\infty$.

Solving this equation gives the density of cohorts for consumers active at time t:

$$\phi(y \mid t) = \alpha e^{-\alpha(t-y)}.$$

□

We now return to the proof of Lemma B.1.

We construct the probability that a L-type consumer who is born at time $y \in [-\infty, t]$ and steals the product is ultimately caught in the time interval $[t-T, t]$. First, consider a L-type consumer from birth cohort $y \leq t - T$ who steals until caught. The probability that this consumer is caught stealing before time t is $1 - e^{-\pi_L(t-y)}$. The probability that this consumer is caught stealing before time $t - T$ is $1 - e^{-\pi_L(t-T-y)}$. So, the probability that this consumer is caught in the time interval $[t-T, t]$ is

$$e^{-\pi_L(t-T-y)} - e^{-\pi_L(t-y)}.$$

Second, consider a L-type consumer from birth cohort $y \in [t-T, t]$ who steals until caught. The probability that this person to be caught in the time interval $[t-T, t]$ is

$$1 - e^{-\pi_L(t-y)}.$$

We now construct the probability that a L-type consumer who is active at time t was caught stealing in the time interval $[t-T, t]$ and therefore are still using the product at the moment t. Recall that exiting the market and being caught stealing are independent events.[2] So, the probability that a L-type consumer who is active at time t was caught stealing in the interval $[t-T, t]$ is:

$$\int_{-\infty}^{t-T} \left[e^{-\pi_L(t-T-y)} - e^{-\pi_L(t-y)}\right] \phi(y \mid t) dy + \int_{t-T}^{t^N} \left[1 - e^{-\pi_L(t-y)}\right] \phi(y \mid t) dy$$

where $\phi(y \mid t)$ is the density of birth cohorts for consumers active at time t. Because $\phi(y \mid t) = \alpha e^{-\alpha(t-y)}$ (see Claim 1), this becomes

$$\begin{aligned} &\int_{-\infty}^{t-T} \left[e^{-\pi_L(t-T-y)} - e^{-\pi_L(t-y)}\right] \alpha e^{-\alpha(t-y)} dy + \int_{t-T}^{t} \left[1 - e^{-\pi_L(t-y)}\right] \alpha e^{-\alpha(t-y)} dy \\ = \; & \frac{\alpha}{\alpha + \pi_L} \left(e^{-\alpha T} - 1\right) + \left(1 - e^{-\alpha T}\right). \end{aligned}$$

[2] The joint probability that the consumer from a birth cohort $y \leq t - T$ is still active at time t and is caught stealing in the range $[t-T, t]$ is $e^{-\alpha(t-y)} \left[e^{-\pi_L(t-T-y)} - e^{-\pi_L(t-y)}\right]$. The joint probability that a type-L consumer from cohort $y \in [t-T, t]$ is active at time t and is caught stealing in $[t-T, t]$ is $e^{-\alpha(t-y)} \left[1 - e^{-\pi_L(t-y)}\right]$.

Denote $q(t;T)$ as the probability that a L-type consumer who is active at time t was caught in the interval $[t-T, t]$. We have:

$$q(t;T) = \frac{\pi_L}{\alpha + \pi_L}(1 - e^{-\alpha T}).$$

Therefore, the probability that a L-type consumer who is active at time t was caught in the interval $(-\infty,\, t-T]$ is

$$q(t;-\infty) - q(t;T) = \frac{\pi_L}{\alpha + \pi_L} e^{-\alpha T}.$$

The number of users at time t is

$$N(T) = \lambda_H + \lambda_L \left(1 - \frac{\pi_L}{\alpha + \pi_L} e^{-\alpha T}\right).$$

■

Using Lemma B.1, the firm's optimization problem simplifies. The firm chooses $\{P, X, T\}$ to maximize its profit:

$$\text{Max}_{\{P,X,T\}} \quad \lambda_H P + \lambda_L \theta_L X$$

subject to

$$N(T)V_H - P \geq N(T)V_H - \theta_H(X + e^{-rT}P)$$
$$(1 - \theta_L e^{-rT})N(T)V_L - \theta_L X \geq 0$$
$$P \in (N(T)V_L, N(T)V_H].$$

The second condition, i.e. the L-type's individual-rationality constraint, must bind. So we have

$$X^* = \left(\frac{1 - \theta_L e^{-rT}}{\theta_L}\right) N(T)V_L.$$

Using this expression, we rewrite the H-type's incentive-compatibility constraint as

$$P \leq M(T)N(T)V_L,$$

where $M(T) \equiv \left(\frac{\theta_H}{1-\theta_H e^{-rT}}\right)\left(\frac{1-\theta_L e^{-rT}}{\theta_L}\right)$ as defined in the baseline model.

We can rewrite the program as

$$\text{Max}_{\{P,T\}} \quad \lambda_H P + \lambda_L \left(1 - \theta_L e^{-rT}\right) N(T)V_L$$

subject to

$$\frac{P}{N(T)} \in (V_L, \min\{M(T)V_L, V_H\}].$$

If $M(T)V_L \geq V_H$, then $P = N(T)V_H$. In this case, the firm's profit increases in T, so the optimal license duration is $T^* = \infty$.

If $M(T)V_L < V_H$, then $P = M(T)N(T)V_L$. The firm's profit becomes

$$N(T)[\lambda_H M(T)V_L + \lambda_L \left(1 - \theta_L e^{-rT}\right) V_L]. \tag{11}$$

Note that the second term, $[\lambda_H M(T)V_L + \lambda_L \left(1 - \theta_L e^{-rT}\right) V_L]$, is the same as the firm's profit in the beseline model without network effects, which is convex in T. As in the baseline model, the second term is maximized by either $T = \infty$ or $T = \widetilde{T}$ such that $M(\widetilde{T})V_L = V_H$. The first term $N(T)$ reflects the network effect and increases in T. Therefore, if the second term is maximized by $T = \infty$, the whole profit function is also maximized by $T = \infty$; if the second term is maximized by $T = \widetilde{T}$, the whole profit function is maximized by $T \geq \widetilde{T}$.

Similar to the analysis in the baseline model, one can show that the firm would not implement any policy under which both consumer types steal the product. Moreover, the firm would not choose a uniform price $P = (\lambda_H + \lambda_L)V_L$ and a large settlement amount X to deter stealing, as it could generate higher profits by allowing piracy and adopting a settlement policy with $T = \infty$, which does not sacrifice network effects but facilitates price discrimination.

Based on the earlier analysis and Proposition 1, we have

Proposition B.1. *(Network effects.) Suppose that network effects exist and property rights are strong* $\left(D > \overline{D}\right)$. *(1) Suppose* $\frac{V_H}{V_L} \leq \frac{\theta_H}{\theta_L}$. *Then* $P^* = N(\infty)V_H$, $X^* = N(\infty)(\overline{D} + V_L)$, *and* $T^* = \infty$. *(2) Suppose* $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$. *If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* = N(T^*)M(T^*)V_L$, $X^* = N(T^*)(\overline{D} + (1 - e^{-rT^*})V_L)$, *and* $T^* \geq \widetilde{T} \in [0, \infty)$; *otherwise* $P^* = N(\infty)\theta_H(\overline{D} + V_L)$, $X^* = N(\infty)(\overline{D} + V_L)$, *and* $T^* = \infty$. *In both (1) and (2), the H-types buy and L-types steal and settle if caught.*

Given network effects, the firm's optimal license duration is (weakly) longer than that in the baseline model. It is possible to have a strictly longer duration, $T^* > \widetilde{T}$. In particular, suppose that $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$ and $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$. If $T = \infty$, then the optimal price is $(\lambda_H + \lambda_L)\theta_H(\overline{D} + V_L) = (\lambda_H + \lambda_L)\frac{\theta_H}{\theta_L}V_L$ and the firm's profit becomes

$$(\lambda_H + \lambda_L)\left[\lambda_H \frac{\theta_H}{\theta_L} + \lambda_L\right] V_L,$$

If $T = \widetilde{T}$, the optimal price is $N(\widetilde{T})V_H$ and the firm's profit is

$$\left[\lambda_H + \lambda_L \left(1 - \frac{\pi_L}{\alpha + \pi_L} e^{-\alpha \widetilde{T}}\right)\right] \left[\lambda_H V_H + \lambda_L \big(1 - \theta_L e^{-r\widetilde{T}}\big) V_L\right],$$

which is arbitrarily close to $\lambda_H \big[\lambda_H \frac{\theta_H}{\theta_L} + \lambda_L\big] V_L$ when α is arbitrarily close to 0 and $\frac{\lambda_L}{\lambda_H}$ is arbitrarily close to $\frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L} - 1\big)$. Therefore, when α is sufficiently small and $\frac{\lambda_L}{\lambda_H}$ is smaller than $\frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L} - 1\big)$ but not too small, the firm can generate higher profits by choosing $T = \infty$ than choosing $T = \widetilde{T}$. In this case, the optimal license duration is strictly longer than $\widetilde{T}$.

Section 4.3. Entry Deterrence

Denote the incumbent as firm I and the entrant as firm E, who arrives according to an exogenous Poisson process with arrival rate β. Upon arrival, the entrant decides whether to enter the market or not. The lump-sum entry cost is $K > 0$. The entrant's product generates an instantaneous value $v_i + u$ for a type-i consumer, where $u > 0$ and we let $U = u/r$. For simplicity, we focus on the scenario where property rights are strong.

Note that the incremental value created by the entrant, U, is the same for both consumer types. So, the prices following entry are $P_I = 0$ for the incumbent and $P_E = U$ for the entrant, and all free consumers buy from the entrant. Conditional upon entering the market, the entrant's market share increases over time. At the moment of entry, none of the H-types are free, as all are under long-term contracts with the incumbent. Over time, the old H-types exit the market and new H-types enter, assuring the entrant a steady stream of H-type consumers. At the moment of entry, a fraction of the L-types are free, generating immediate sales for the entrant. Over time, as the licenses included in settlement contracts expire and new L-type consumers enter the market, the sales to the L-types gradually increase and then, after duration T has passed, the stream of the L-type consumers stabilizes.

Lemma B.2. *Suppose that property rights are strong* $\big(D > \overline{D}\big)$. *The entrant's (discounted) sales volume at the moment of entry is*

$$Q(T) = \lambda_H \frac{\alpha}{\rho} + \lambda_L \frac{r}{\rho} \delta(T)$$

where $\delta(T)$ *is the percentage of the L-types who are free.* $Q(T)$ *strictly decreases in* T *and* $Q(0) = \lambda_H \frac{\alpha}{\rho} + \lambda_L \frac{r}{\rho}$.

Proof of Lemma B.2. Suppose that the firm has been using settlement contracts with license duration T prior to entry. Suppose that the entrant enters the market at time t^E. There is a proportion λ_H of high types, none of whom are free. There is a proportion λ_L of low types. The ones who were caught in the interval $[t^E - T, t^E]$ are locked in, but the rest are free to purchase from the entrant at time t^E. As shown in the proof of Proposition B.1, the probability that an L-type consumer who is active at time t^E was caught in the interval $[t^E - T, t^E]$ is:

$$q(t^E; T) = \frac{\pi_L}{\alpha + \pi_L}(1 - e^{-\alpha T}).$$

More generally, given any $t \in [t^E, t^E + T]$, the conditional probability that an L-type consumer who is alive at time t was previously caught in the interval $[t - T, t^E]$ is:

$$\int_{-\infty}^{t-T} \left[e^{-\pi_L(t-T-y)} - e^{-\pi_L(t^E - y)}\right] \alpha e^{-\alpha(t-y)} dy + \int_{t-T}^{t^E} \left[1 - e^{-\pi_L(t^E - y)}\right] \alpha e^{-\alpha(t-y)} dy$$

which simplifies to

$$q(t; T) = \frac{\pi_L}{\alpha + \pi_L}(e^{-\alpha(t - t^E)} - e^{-\alpha T}).$$

Finally, for any $t > t^E + T$, none of the L-type consumers are locked-in.

We now construct the discounted mass of the L-types who are "free" at the time of E's entry. Recall that the fraction of the L-types at any moment of time t is λ_L. At any moment $t \in [t^E, t^E + T]$, since $q(t; T)$ increases in T, the fraction of "free"consumers among the L-types, $\lambda_L[1 - q(t; T)]$, is decreasing in T.

Normalizing t^E to be 0, the discounted mass of the L-types who are free at the time of entry is

$$\lambda_L \int_0^\infty [1 - q(t; T)] \, e^{-\rho t} dt$$

$$= \lambda_L \left[\int_0^\infty e^{-\rho t} dt - \int_0^T \left(\frac{\pi_L}{\alpha + \pi_L}\right)(e^{-\alpha t} - e^{-\alpha T}) e^{-\rho t} dt\right]$$

$$= \lambda_L \left(\frac{(\alpha + \pi_L)(\alpha + \rho) - \rho\pi_L - \alpha\pi_L e^{-(\alpha+\rho)T} + \pi_L(\alpha + \rho)e^{-\alpha T}}{\rho(\alpha + \pi_L)(\alpha + \rho)}\right)$$

Because $r = \alpha + \rho$, we can rewrite this as

$$\lambda_L \left(\frac{r(\alpha + \pi_L) - \rho\pi_L + r\pi_L e^{-\alpha T} - \alpha\pi_L e^{-rT}}{\rho r(\alpha + \pi_L)}\right). \tag{12}$$

Additionally, note that the H-types always purchase the incumbent's product before the entrant arrives. Thus, all these consumers are locked in at the moment when the entrant arrives. However, over time when these consumers exit from the market, the new generations of the H-types are free. Note that, among those consumers who are active at $t \in [0, \infty)$, the probability for them to be born before moment 0 (i.e. the moment of entry) is

$$\int_{-\infty}^{0} \phi(y \mid t) dy = \int_{-\infty}^{0} \alpha e^{-\alpha(t-y)} dy = e^{-\alpha t}.$$

Thus, at the moment when the entrant arrives, the discounted mass of future H-types who are free is

$$\lambda_H \int_0^{\infty} \left[1 - e^{-\alpha t}\right] e^{-\rho t} dt = \lambda_H \frac{\alpha}{\rho(\alpha + \rho)}. \tag{13}$$

We can now present a closed-form expression for the discounted mass of free consumers at the time of E's entry into the market. Adding the discounted mass of free L-types in equation (12) and the discounted mass of free H-types in (13), and remembering that $r = \alpha + \rho$, we have

$$\lambda_H \frac{\alpha}{\rho r} + \lambda_L \left(\frac{\alpha(r + \pi_L) + r\pi_L e^{-\alpha T} - \alpha \pi_L e^{-rT}}{\rho r(\alpha + \pi_L)} \right) \tag{14}$$

which decreases in T. Multiplying (14) by $r = \alpha + \rho$ gives the entrant's discounted sales volume:[3]

$$Q(T) = \lambda_H \frac{\alpha}{\rho} + \lambda_L \left(\frac{\alpha(r + \pi_L) + r\pi_L e^{-\alpha T} - \alpha \pi_L e^{-rT}}{\rho(\alpha + \pi_L)} \right).$$

Substituting $\pi_L = \frac{\theta_L}{1-\theta_L} r$ and rearranging terms gives

$$Q(T) = \lambda_H \frac{\alpha}{\rho} + \lambda_L \frac{r}{\rho} \left(\frac{\alpha + \theta_L (r - \alpha e^{-\rho T}) e^{-\alpha T}}{\alpha + \theta_L \rho} \right) = \lambda_H \frac{\alpha}{\rho} + \lambda_L \frac{r}{\rho} \delta(T).$$

where $\delta(T) = \frac{\alpha + \theta_L (r - \alpha e^{-\rho T}) e^{-\alpha T}}{\alpha + \theta_L \rho}$ is the proportion of "free consumers" among all the L-type consumers. $\delta(T)$ decreases in T and $\delta(0) = 1$. ■

This expression $Q(T) = \lambda_H \frac{\alpha}{\rho} + \lambda_L \frac{r}{\rho} \delta(T)$ can be understood intuitively. Consider the first term on the right-hand side. If no consumer were locked in, all consumers in the over-lapping generations would buy from the entrant, so that

[3] Recall that the entrant sells perpetual licenses to free consumers at price $U = u/r$, where u is the incremental instantaneous value from the entrant's product.

the entrant's (discounted) sales volume to the H-types would be $\lambda_H \frac{\alpha+\rho}{\rho}$. However, when the entrant arrives, all existing H-types (with the total mass λ_H) are locked in. Therefore, the entrant's sales volume to the over-lapping generations of H-types is $\lambda_H \frac{\alpha+\rho}{\rho} - \lambda_H$. The second term on the right-hand side are the discounted sales to the L-types. If $T = 0$, no L-type consumer is locked in, so the entrant's (discounted) sales to the over-lapping generations of the L-types is $\lambda_L \frac{\alpha+\rho}{\rho}$. A license with longer duration T reduces the number of "free" consumers and therefore reduces the entrant's sales volume.

If firm E sinks the entry cost, its expected revenue from the over-lapping generations of consumers is $UQ(T)$. When $K \geq UQ(0)$, E never enters the market; when $K < UQ(\infty)$, E always enters. We shall focus on the more interesting scenario where $K \in [UQ(\infty), UQ(0))$. Given $K \in [UQ(\infty), UQ(0))$, there exists a unique $\overline{T} \in (0, \infty)$ such that

$$UQ(\overline{T}) \equiv K.$$

If the incumbent offers a license of duration $T \geq \overline{T}$ in the settlement offer, entry is deterred; otherwise the entrant (once arrived) will enter the market. Since $\overline{T}$ depends on $\frac{\lambda_L}{\lambda_H}$ but is independent of $\frac{V_H}{V_L}$, we shall consider different ranges of $\frac{V_H}{V_L}$ in the following analysis.

As shown in the baseline model, if property rights are strong and there is no entry threat, the incumbent's optimal policy has the following properties: (1) If $\frac{V_H}{V_L} \leq \frac{\theta_H}{\theta_L}$ or $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$ but $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L} - 1\big)$, then $T^* = \infty$; (2) if $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$ but $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L} - 1\big)$ then $T^* = \widetilde{T} \in [0, \infty)$. It can be verified that $\widetilde{T}$ decreases in $\frac{V_H}{V_L}$. So there exists a unique value $\Delta_1 \in \big(\max\big\{\frac{\theta_H}{\theta_L}, 1 + \frac{\lambda_L}{\lambda_H}\frac{\theta_L}{\theta_H}\big\}, \frac{\pi_H}{\pi_L}\big)$ such that, $T^* = \widetilde{T} \geq \overline{T}$ if and only if $\frac{V_H}{V_L} \leq \Delta_1$. That is, for any $\frac{V_H}{V_L} \leq \Delta_1$, the incumbent can deter entry by just choosing the optimal license duration as if there were no entry threat. The following lemma shows that, when $\frac{V_H}{V_L}$ is larger than Δ_1 but smaller than a certain threshold, the incumbent chooses either $T = \overline{T}$ or $T = \infty$ to deter entry.

Lemma B.3. *Suppose that property rights are strong* $\big(D > \overline{D}\big)$. *Given the other parameter values, there exist three cut-offs* $\Delta_1 < \Delta_2 \leq \Delta_3$ *such that the incumbent deters entry if and only if* $\frac{V_H}{V_L} < \Delta_3$. *(1) If* $\frac{V_H}{V_L} \leq \Delta_1$, *the incumbent chooses the license duration* T^* *as if there were no entry threat (defined in Proposition 1). (2) If* $\frac{V_H}{V_L} \in (\Delta_1,\Delta_2)$ *the incumbent chooses* $T = \overline{T} > T^*$. *(3) If* $\frac{V_H}{V_L} \in [\Delta_2,\Delta_3)$ *the incumbent chooses* $T = \infty > T^*$.[4]

[4]In particular, $\Delta_2 = \Delta_3$ or $[\Delta_2,\Delta_3)$ degenerates to be empty when $\frac{\lambda_L}{\lambda_H} \leq \frac{\theta_H}{\theta_L}(\frac{\theta_H}{\theta_L} - 1)$.

Proof of Lemma B.3. Part (1) follows directly from the text. Now consider parts (2) and (3). Denote the moment for E to arrive as t^E, which follows the distribution $\Phi(t^E) = 1 - e^{-\beta t^E}$. As we show in the baseline model, given any $T > T^*$, the H-types' incentive-compatibility constraint holds so that they purchase the product, while the L-types steal it. That is, choosing a longer duration $T > T^*$ does not change consumers' behaviour before entry. The firm's profit $S^L(T)$ is convex in T. Given $\overline{T} > T^*$, if the incumbent intends to deter entry, it should choose either $T = \overline{T}$ or $T = \infty$. Thus, if the incumbent chooses to deter entry, its profit per consumer is $\max\{S^L(\overline{T}), S^L(\infty)\}$.

If the incumbent chooses T^* and does not deter entry, its per-moment profit before entry occurs is $rS^L(T^*) = (\alpha+\rho)S^L(T^*)$, while its profit after entry occurs becomes 0. The total profit is

$$\int_0^\infty \left[\int_0^z ((\alpha + \rho)S(T^*)e^{-\rho\tau})\alpha e^{-\alpha\tau} d\tau\right] \beta e^{-\beta z} dz = \frac{r}{r+\beta} S^L(T^*).$$

Therefore, the incumbent chooses either $T = \overline{T}$ or $T = \infty$ to deter entry if and only if

$$\frac{r}{r+\beta} S^L(T^*) < \max\{S^L(\overline{T}), S^L(\infty)\}, \tag{15}$$

or equivalently,

$$\frac{r}{r+\beta} \frac{S^L(T^*)}{V_L} < \max\left\{\frac{S^L(\overline{T})}{V_L}, \frac{S^L(\infty)}{V_L}\right\},$$

Note that both $\frac{S^L(\overline{T})}{V_L}$ and $\frac{S^L(\infty)}{V_L}$ are independent of $\frac{V_H}{V_L}$, while T^* decreases in $\frac{V_H}{V_L}$ (given $\frac{V_H}{V_L} > \Delta_1$). When $\frac{V_H}{V_L}$ is arbitrarily close to Δ_1, T^* is arbitrarily close to $\overline{T}$, so that condition (15) holds. By continuity, there exists a unique value $\Delta_3 > \Delta_1$ such that the incumbent deters entry by choosing either $T = \overline{T}$ or $T = \infty$ if and only if $\frac{V_H}{V_L} \in (\Delta_1, \Delta_3)$. When $\frac{V_H}{V_L}$ is larger than but arbitrarily close to Δ_1, T^* is arbitrarily close to $\overline{T}$. Since $S^L(T^*) > S^L(\infty)$ when $\frac{V_H}{V_L} > \Delta_1 > \max\left\{\frac{\theta_H}{\theta_L}, 1 + \frac{\lambda_L}{\lambda_H}\frac{\theta_L}{\theta_H}\right\}$, there exists a unique value $\Delta_2 \in (\Delta_1, \Delta_3]$ such that $S^L(\overline{T}) > S^L(\infty)$ if and only if $\frac{V_H}{V_L} < \Delta_2$.

Finally, if $\frac{\lambda_L}{\lambda_H} \leq \frac{\theta_H}{\theta_L}\left(\frac{\theta_H}{\theta_L} - 1\right)$, it can be verified that $\frac{V_H}{V_L} > \frac{\theta_H}{\theta_L}$ and $S^L(\infty) < S^L(T)$ for any $T \geq T^*$. In this case, we must have $\Delta_2 = \Delta_3$. ■

When facing an entry threat, the incumbent has incentives to extend the duration of the license included in settlement contracts. A natural question is whether and when imposing a cap on the license duration can raise social welfare. Consider two cases. First, when $\frac{V_H}{V_L} \in (\Delta_1, \Delta_2)$, as shown in Lemma B.3, the incumbent finds it optimal to deter entry by choosing $\overline{T} > T^*$. A slightly shorter duration $T' = \overline{T} - \varepsilon$ would accommodate entry and strictly increase social welfare. Note that, given $T = \overline{T} - \varepsilon$, upon entry, each type-$i$ consumer who is free gets a net surplus of V_i, $i = H, L$. A cap $\overline{T} - \varepsilon$ on the license duration reduces the welfare before entry as stealing consumers' consumption duration drops by ε; but it raises the welfare after entry by allowing all "free" consumers to get a net surplus and allowing the entrant to get a profit of $Q(\overline{T} - \varepsilon)u - K$. When ε is arbitrarily close to 0, the entrant's profit and the welfare loss before entry are both negligible. However, the net surplus for all free consumers is strictly positive. Therefore, if the incumbent finds it optimal to deter entry by choosing $\overline{T}$, an alternative policy with a shorter license duration is more efficient.

Second, when $\Delta < \Delta_2$, the incumbent finds it optimal to deter entry by choosing either T^* or $\overline{T}$. If the firm is not allowed to include licenses in settlement contracts so that E would enter the market, on one hand, stealing consumers' consumption duration drops before entry occurs; on the other hand, upon entry, all "free" consumers get a net surplus and the entrant generates a profit of $UQ(0) - K$. The total welfare effect is ambiguous. However, if both T^* and $\overline{T}$ are sufficiently small, the welfare loss before entry becomes arbitrarily small, while the free consumers' net surplus is strictly positive. In this case, settlement without licensing is more efficient than settlement with licensing.

Proposition B.2. *(Entry Deterrence.) (1) If* $\frac{V_H}{V_L} \in (\Delta_1, \Delta_2)$*, the incumbent combines the settlement contract with a license of duration* $\overline{T} > T^*$ *to deter entry. There exists an alternative settlement policy with* $T' < \overline{T}$*, which accommodates entry and strictly increases social welfare. (2) There exists a cut-off* $\widehat{K} < UQ(0)$ *such that, given any* $K \in (\widehat{K}, UQ(0)]$*, there exists a value* $\widehat{\Delta} < \Delta_2$ *such that settlement without licensing* ($T = 0$) *is more efficient than settlement with licensing when* $\frac{V_H}{V_L} \in (\widehat{\Delta}, \Delta_2]$.

To summarize, settlement with licensing can create anti-competitive effects. Imposing a cap on the license duration in settlement offers or forbidding firms from combining settlement contracts with licenses may increase welfare.

Section 4.4. Recidivism

For simplicity, we focus on the case with strong property rights. We also maintain Assumption 1 $\left(\frac{v_H}{\pi_H} \leq \frac{v_L}{\pi_L}\right)$. If a type-$i$ consumer steals the product, then after settling with the firm and finishing the license of duration T, the consumer should have incentives to steal again. Recall that τ is the moment for the consumer to be caught for the first time. The consumer uses the product till moment $\tau + T$ and pays the settlement amount X at moment τ; at moment $\tau + T$, the consumer will steal the product again. Thus, the consumer's expected net value from stealing is

$$\widetilde{B}_i = \int_0^\infty \left\{ \int_0^{\tau+T} v_i e^{-rt} dt - X e^{-r\tau} + \widetilde{B}_i e^{-r(\tau+T)} \right\} \pi_i e^{-\pi_i \tau} d\tau.$$

This simplifies to

$$\widetilde{B}_i = V_i - \frac{\theta_i X}{1 - \theta_i e^{-rT}}.$$

If an L-type steals the product at every opportunity, then his gross value is V_L. Therefore, the firm's profit from this consumer is $V_L - \widetilde{B}_L$, or equivalently, $\frac{\theta_L X}{1-\theta_L e^{-rT}}$.

The following proposition shows that there exists a continuum of policies that can implement perfect price discrimination, under which the H-types buy and the L-types steal the product repeatedly. By assumption, when choosing among these revenue-equivalent policies, the firm chooses the policy with the longest license duration T (and therefore the least amount of stealing in equilibrium).[5] Recall that $\widetilde{T}$ is defined by $M(\widetilde{T})V_L \equiv V_H$ in the baseline model.

Proposition B.3. *(Recidivism.) Suppose that recidivism is feasible and property rights are strong* $\left(D > \overline{D}\right)$*. Under the optimal policy, the H-types buy and the L-types steal the product repeatedly and settle if caught.*

1. *If* $\frac{V_H}{V_L} \leq \frac{\theta_H}{\theta_L}$ *then there exists a continuum of policies extracting full consumer surplus,* $P^* = V_H$ *and* $X^* = \overline{D} + (1 - e^{-rT})V_L$ *for any* $T \geq 0$*. The firm chooses the policy with* $T = \infty$*.*

2. *If* $\frac{V_H}{V_L} \in (\frac{\theta_H}{\theta_L}, \frac{\pi_H}{\pi_L}]$ *there exists a continuum of policies extracting full consumer surplus,* $P^* = V_H$ *and* $X^* = \overline{D} + (1 - e^{-rT})V_L$ *for any* $T \in [0, \widetilde{T}]$*. The firm chooses the policy with* $T = \widetilde{T}$*.*

Proof of Proposition B.3. The firm chooses $P \in (V_L, V_H]$, X, and T to maximize its profits subject to the H-types' incentive-compatibility constraint and the L-types' individual-rationality constraint,

$$\text{Max}_{\{P,X,T\}} \quad \lambda_H P + \lambda_L \frac{\theta_L X}{1 - \theta_L e^{-rT}}$$

subject to

$$P \leq \frac{\theta_H X}{1 - \theta_H e^{-rT}}$$

[5] This tie-breaking assumption is consistent with positive costs of litigation and settlement.

$$\frac{\theta_L X}{1-\theta_L e^{-rT}} \le V_L$$
$$P \in (V_L, V_H].$$

The L-types' individual-rationality constraint must bind. So we have

$$X^* = \frac{1-\theta_L e^{-rT}}{\theta_L} V_L = \overline{D} + (1-e^{-rT)}V_L.$$

The firm's program can be re-written as

$$\text{Max}_{\{P,X,T\}} \quad \lambda_H P + \lambda_L V_L$$

subject to

$$V_L < P \le \min\{M(T)V_L, V_H\}$$

Note that $M(T)$ decreases in T, with $M(0) = \frac{\pi_H}{\pi_L}$ and $M(\infty) = \frac{\theta_H}{\theta_L}$. Consider two cases.

If $\frac{V_H}{V_L} \le \frac{\theta_H}{\theta_L}$, then for any T, $M(T)V_L \ge V_H$. In this case, the firm's optimal price is $P^* = V_H$ and the firm gets the same profit $\lambda_H V_H + \lambda_L V_L$ given any $T \ge 0$. The firm implements perfect price discrimination and extracts all consumer surplus.

If $\frac{V_H}{V_L} \in \left(\frac{\theta_H}{\theta_L}, \frac{\pi_H}{\pi_L}\right]$, as defined in the baseline model, there exists a unique $\widetilde{T} \in [0,\infty)$ such that $M(\widetilde{T})V_L \equiv V_H$. Then for any $T \le \widetilde{T}$, $M(T)V_L \ge V_H$. In this case, again the firm's optimal price is $P^* = V_H$ and the firm gets the same profit $\lambda_H V_H + \lambda_L V_L$ given any $T \le \widetilde{T}$. ■

Section 4.5. Concave Detection Rate

In the baseline model, we assume $\frac{v_H}{\pi_H} \le \frac{v_L}{\pi_L}$. In this subsection, suppose that $\frac{v_H}{\pi_H} > \frac{v_L}{\pi_L}$ and property rights are strong $\left(D > \overline{D}\right)$.

Since the H-type has a much higher consumption value than the L-type, different from the baseline model, the firm may have incentives to refuse settlement and sell the product only to the H-type. This is true when there are relatively few L-types in the consumer population ($\frac{\lambda_L}{\lambda_H}$ is small). However, when $\frac{\lambda_L}{\lambda_H}$ is not small, the firm still takes a "soft" stance with pirates and uses settlement for price discrimination. Similar to the discussion in the baseline model, since $\frac{V_H}{V_L} > \frac{\pi_H}{\pi_L} > \frac{\theta_H}{\theta_L}$, perfect price discrimination is impossible, that is, the firm could not offer the policy $\{P = V_H,\, X = \overline{D} + V_L,\, T = \infty\}$ to extract all consumer surplus. Instead, the firm either reduces the market price or shortens the duration of the license in the settlement contract. The following proposition characterizes the firm's optimal policy.

Proposition B.4. *(Concave Detection Rate.) Suppose* $\frac{v_H}{\pi_H} > \frac{v_L}{\pi_L}$ *and property rights are strong* $\left(D > \overline{D}\right)$. *The firm's optimal policy has the following properties.*

1. *If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L}\frac{\pi_L}{\pi_H} - 1\right)$ *then* $P^* = V_H$ *and the firm refuses to settle lawsuits. No consumer steals the product.*
2. *If* $\frac{\lambda_L}{\lambda_H} \in \left[\frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L}\frac{\pi_L}{\pi_H} - 1\right), \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)\right)$ *then* $P^* = V_H$, $X^* = \overline{D}$ *and* $T^* = 0$. *Both types steal and settle if caught.*
3. *If* $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$ *then* $P^* = \theta_H(\overline{D} + V_L)$, $X^* = \overline{D} + V_L$ *and* $T^* = \infty$. *The H-types buy and the L-types steal and settle if caught.*

Proof of Proposition B.4. We first show two claims. Claim 2 characterizes the optimal policy under which the H-types buy and the L-types steal the product, and Claim 3 characterize the optimal policy under which both types steal the product.

Claim 2. *Suppose that the H-types buy and the L-types steal the product. (1) If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$ *then the optimal policy satisfies* $P = \frac{\pi_H}{\pi_L}V_L$, $X = \overline{D}$ *and* $T = 0$. *(2) If* $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$, *the optimal policy satisfies* $P = \theta_H(\overline{D} + V_L)$, $X = \overline{D} + V_L$ and $T = \infty$.

Proof of Claim 2. Suppose $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$. Similar to the analysis in the baseline model, one can show that the optimal policy satisfies $X = \overline{D} + (1 - e^{-rT})V_L$ and $P \in (V_L, \min\{M(T)V_L, V_H\}]$. Given $\frac{V_H}{V_L} > \frac{\pi_H}{\pi_L}$, we have $M(T)V_L \leq M(0)V_L = \frac{\pi_H}{\pi_L}V_L < V_H$ for any $T \geq 0$. Therefore, any $T \geq 0$ is feasible. The firm's profit $S^L(T)$ is convex and therefore maximized by either $T = 0$ or $T = \infty$. It is easy to verify that $S^L(0) > S^L(\infty)$ if and only if $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$. The result then follows. □

Claim 3. *Suppose that both types steal the product. (1) If* $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$, *then the optimal policy satisfies* $P = V_H$, $X = \overline{D}$, and $T = 0$. *(2) If* $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$, *the optimal policy satisfies* $P = V_H$, $X = \overline{D} + V_L$ *and* $T = \infty$ *when* $\frac{V_H}{V_L} \in \left(\frac{\pi_H}{\pi_L}, 1 + \frac{\lambda_L}{\lambda_H}\frac{\theta_L}{\theta_H}\right]$, *and* $P = V_H$, $X = \overline{D}$ *and* $T = 0$ *when* $\frac{V_H}{V_L} > 1 + \frac{\lambda_L}{\lambda_H}\frac{\theta_L}{\theta_H}$.

Proof of Claim 3. Similar to the analysis in the baseline model, if both types steal the product, the firm's program can be written as

$$\text{Max}_{(T)} S^B(T) = \lambda_H\theta_H\left[\left(\frac{1 - \theta_L e^{-rT}}{\theta_L}\right)V_L + e^{-rT}V_H\right] + \lambda_L(1 - \theta_L e^{-rT})V_L,$$

subject to

$$V_H > M(T)V_L.$$

Given $\frac{V_H}{V_L} > \frac{\pi_H}{\pi_L}$, we have $M(T)V_L \leq M(0)V_L = \frac{\pi_H}{\pi_L}V_L < V_H$ for any $T \geq 0$. Therefore, any $T \geq 0$ is feasible. Note that $S^B(T)$ is monotonic in T. Therefore, $S^B(T)$ is maximized by either $T = 0$ or $T = \infty$. One can verify that $S^B(\infty) < S^B(0)$ if and only if $\frac{V_H}{V_L} > 1 + \frac{\theta_L}{\theta_H}\frac{\lambda_L}{\lambda_H}$. If $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$, then $1 + \frac{\theta_L}{\theta_H}\frac{\lambda_L}{\lambda_H} < \frac{\pi_H}{\pi_L} < \frac{V_H}{V_L}$, so $S^B(\infty) < S^B(0)$. If $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{\pi_H}{\pi_L} - 1\right)$, then $1 + \frac{\theta_L}{\theta_H}\frac{\lambda_L}{\lambda_H} \geq \frac{\pi_H}{\pi_L}$. Thus, if $\frac{V_H}{V_L} \in \left(\frac{\pi_H}{\pi_L}, 1 + \frac{\theta_L}{\theta_H}\frac{\lambda_L}{\lambda_H}\right]$, $S^B(\infty) \geq S^B(0)$; if $\frac{V_H}{V_L} > 1 + \frac{\theta_L}{\theta_H}\frac{\lambda_L}{\lambda_H}$, $S^B(\infty) < S^B(0)$. $\square$

We now return to the proof of Proposition B.4.

(1) Suppose $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)$. If only the L-types steal the product, as shown in Claim 2, the optimal policy has $P = \theta_H(\overline{D} + V_L)$, $T = \infty$, and $X = \overline{D} + V_L$, with the firm's profit as $S^L(\infty)$. If both types steal the product, as shown in Claim 3, the optimal policy has $P = V_H$, $T = \infty$, and $X = \overline{D} + V_L$, with the firm's profit as $S^B(\infty)$. Note that, $S^B(\infty) = S^L(\infty)$, which is greater than the firm's profit if stealing is fully deterred. By assumption, the firm prefers the policy under which only the L-type steal the product.

(2) Suppose $\frac{\lambda_L}{\lambda_H} \in \left(\frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L}\frac{\pi_L}{\pi_H} - 1\right), \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L} - 1\right)\right]$. If only the L-types steal the product, as shown in Claim 2, the optimal policy has either $T = \infty$ or $T = 0$. If both types steal the product, as shown in Claim 3, the optimal policy has $P = V_H$, $T = 0$, and $X = \overline{D}$, leading to firm profits of $S^B(0)$. Given $\frac{V_H}{V_L} > 1 + \frac{\theta_L}{\theta_H}\frac{\lambda_L}{\lambda_H}$, we have

$$\begin{aligned} S^B(0) &= \lambda_H\theta_H\left[\left(\frac{1-\theta_L}{\theta_L}\right)V_L + V_H\right] + \lambda_L(1-\theta_L)V_L \\ &> \lambda_H\frac{\theta_H}{\theta_L}V_L + \lambda_L V_L = S^L(\infty) \\ &> (\lambda_H + \lambda_L)V_L. \end{aligned}$$

Furthermore, given $\frac{V_H}{V_L} > \frac{\pi_H}{\pi_L}$, we have

$$\begin{aligned} S^B(0) &= \lambda_H\theta_H\left[\left(\frac{1-\theta_L}{\theta_L}\right)V_L + V_H\right] + \lambda_L(1-\theta_L)V_L \\ &> \lambda_H\frac{\pi_H}{\pi_L}V_L + \lambda_L(1-\theta_L)V_L = S^L(0). \end{aligned}$$

Finally, if and only if $\frac{\lambda_L}{\lambda_H} \geq \frac{\theta_H}{\theta_L}\left(\frac{V_H}{V_L}\frac{\pi_L}{\pi_H} - 1\right)$, we have

$$S^B(0) = \lambda_H\theta_H\left[\left(\frac{1-\theta_L}{\theta_L}\right)V_L + V_H\right] + \lambda_L(1-\theta_L)V_L \geq \lambda_H V_H.$$

Therefore, if $\frac{\lambda_L}{\lambda_H} \in \Big[\frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L}\frac{\pi_L}{\pi_H} - 1\big), \frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L} - 1\big)\Big)$, the firm's profit is maximized by $P^* = V_H$, $X^* = \overline{D}$, and $T^* = 0$, under which both types steal.

(3) Suppose $\frac{\lambda_L}{\lambda_H} < \frac{\theta_H}{\theta_L}\big(\frac{V_H}{V_L}\frac{\pi_L}{\pi_H} - 1\big)$. Similar to the analysis in part (2), we have $S^B(0) > S^L(0) \geq S^L(\infty) > (\lambda_H + \lambda_L)V_L$ and $S^B(0) < \lambda_H V_H$. Therefore, in this case, the firm chooses $P^* = V_H$ and $X > \overline{D} + V_L$ to deter stealing. ■